EE 232 Lightwave Devices
Lecture 2: Basic Concepts of Lasers

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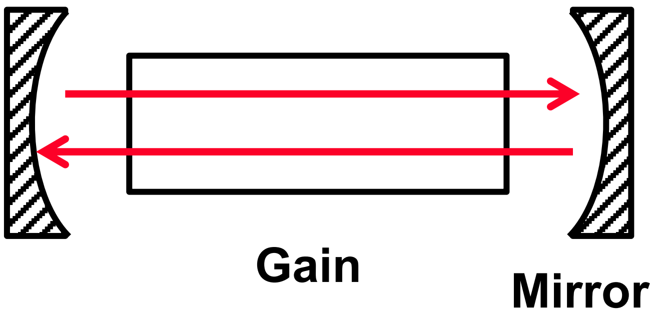
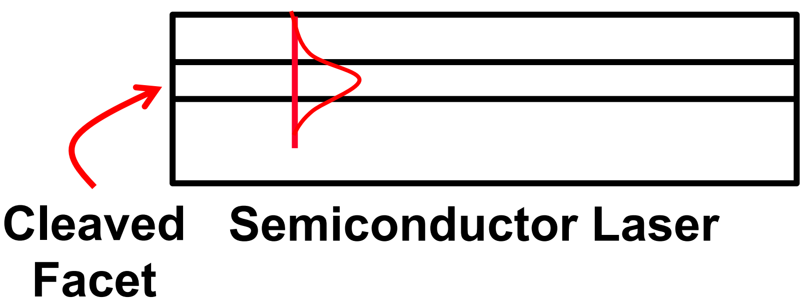
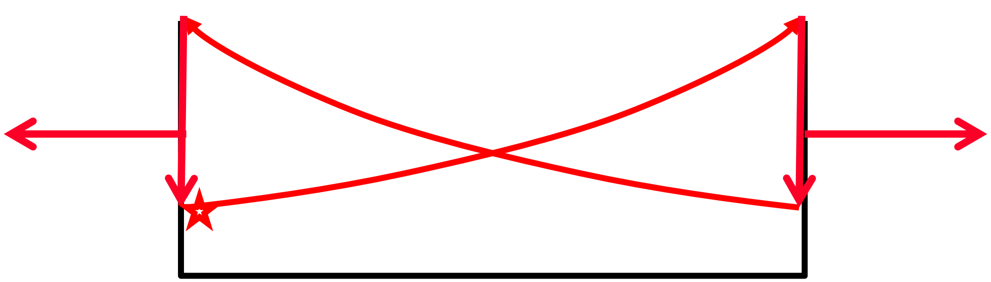
Basic Concept of Lasers

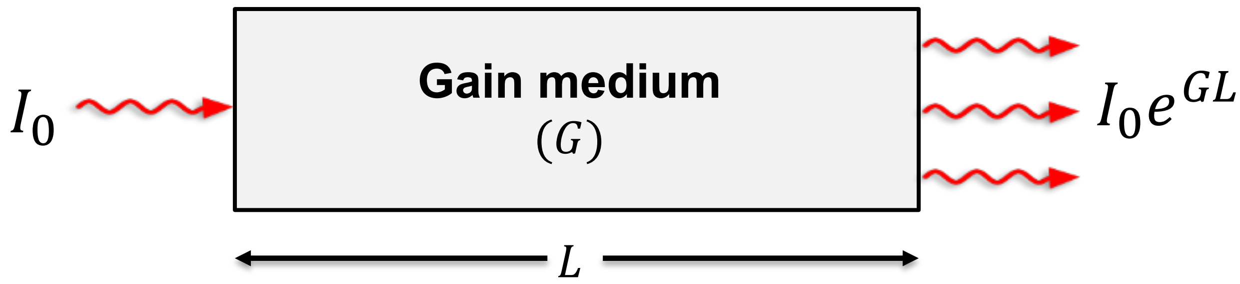






- Laser:
  - Light Amplification by Stimulated Emission of Radiation
- Basic elements:
  - Gain media
  - Optical cavity
- Threshold condition:
  - Bias point where laser starts to "lase"
  - Gain (nearly) equals loss

Gain Medium

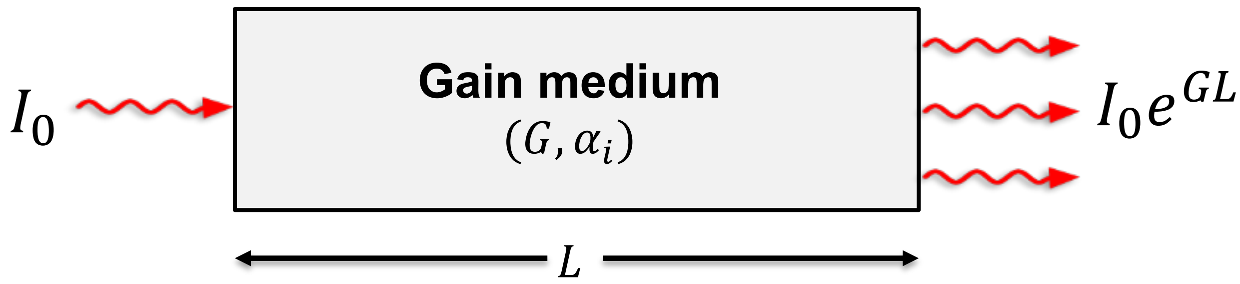


$$
\begin{aligned}
\Delta I &= I(z+\Delta L) - I(z) \\
&= I(z) + I(z)G\Delta L - I(z) \\
&= I(z)G\Delta L
\end{aligned}
$$

$$G = \frac{\Delta I}{I}\frac{1}{\Delta L}$$

Gain is the fractional increase in light intensity per unit length (Units are cm^{-1})

Gain Medium (with internal loss)

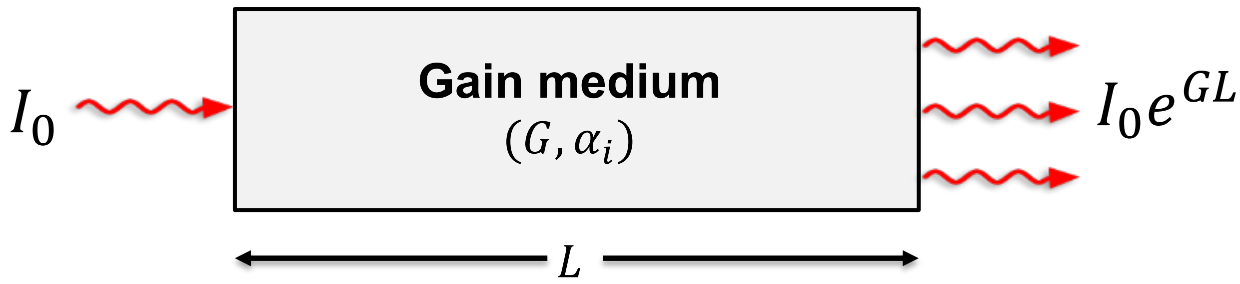


$$\Delta I = I(z+\Delta L) - I(z)$$
$$= I(z) + I(z)(G-\alpha_i)\Delta L - I(z)$$
$$= I(z)(G-\alpha_i)\Delta L$$

$$(G-\alpha_i) = \frac{\Delta I}{I}\frac{1}{\Delta L}$$

Internal loss (α_i) is the fractional decrease in light intensity per unit length (unrelated to fundamental absorption) (Units are cm^{-1})

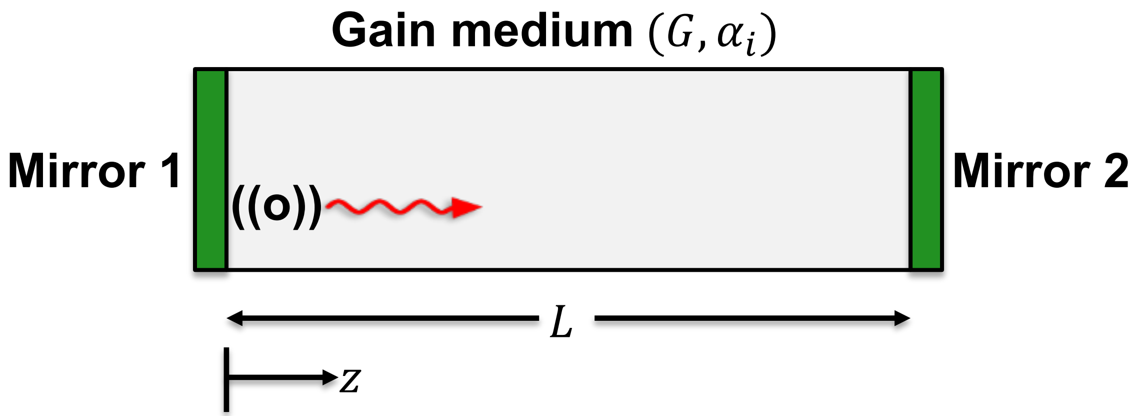
Gain Medium (with internal loss)



$$(G - \alpha_i) = \frac{\Delta I}{I}\frac{1}{\Delta L} \rightarrow \frac{dI}{dz}\frac{1}{I}$$

$$I(z) = I_0 e^{(G-\alpha_i)z}$$

Gain with cavity

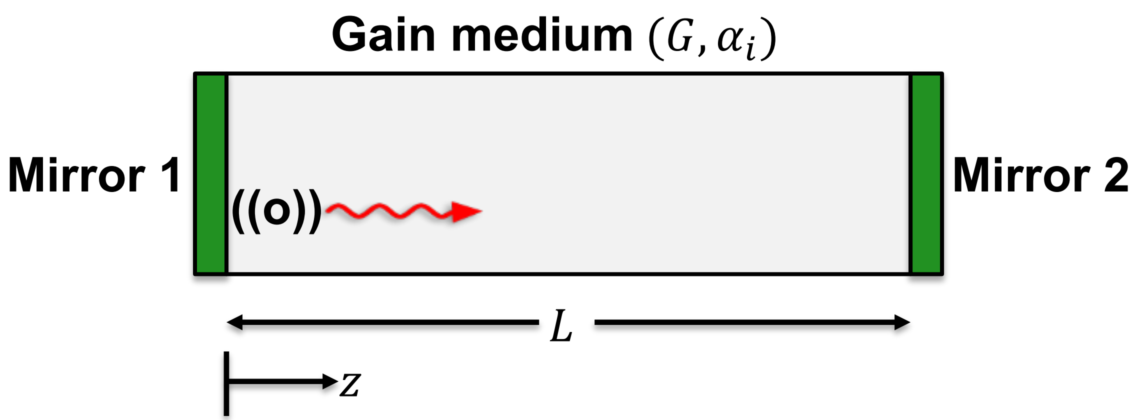


$$1 = \frac{I(z=0^+ + 2L)}{I(z=0^+)} = e^{2(G_{th}-\alpha_i)L} R_2 R_1$$

Threshold condition for self-sustaining oscillation

Round-trip gain

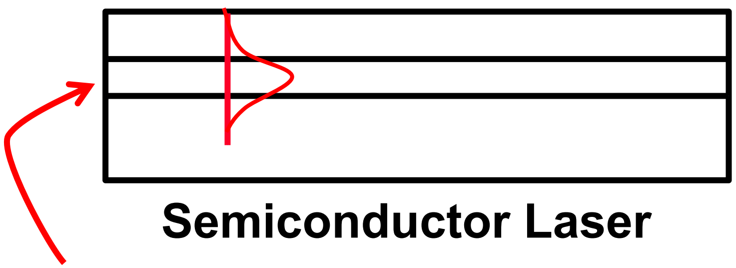
Gain with cavity



$$G_{th} = \frac{1}{2L}\ln\left(\frac{1}{R_2 R_1}\right) + \alpha_i$$
$$= \alpha_m + \alpha_i$$

Threshold condition for self-sustaining oscillation

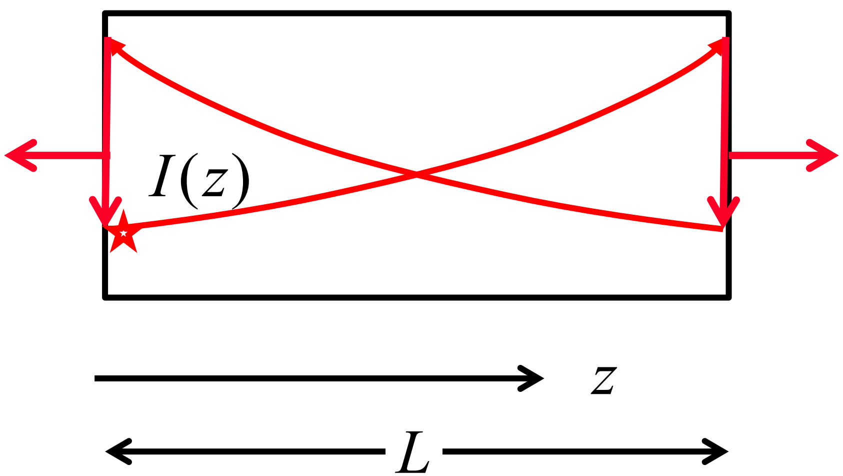
"Edge-Emitting" Semiconductor Lasers



Semiconductor Laser

Cleaved Facet

$$R = \left(\frac{n-1}{n+1}\right)^2, \quad R \sim 30\% \text{ for } n = 3.5$$



g : gain coefficient [cm^{-1}]

Light amplfication: $I(z) = I_0 e^{\Gamma g z}$

Γ: confinement factor

(fraction of energy in gain media)

Threshold condition:

Round-trip gain = 1

$$e^{\Gamma g L - \alpha_i L} R_1 e^{\Gamma g L - \alpha_i L} R_2 = 1$$

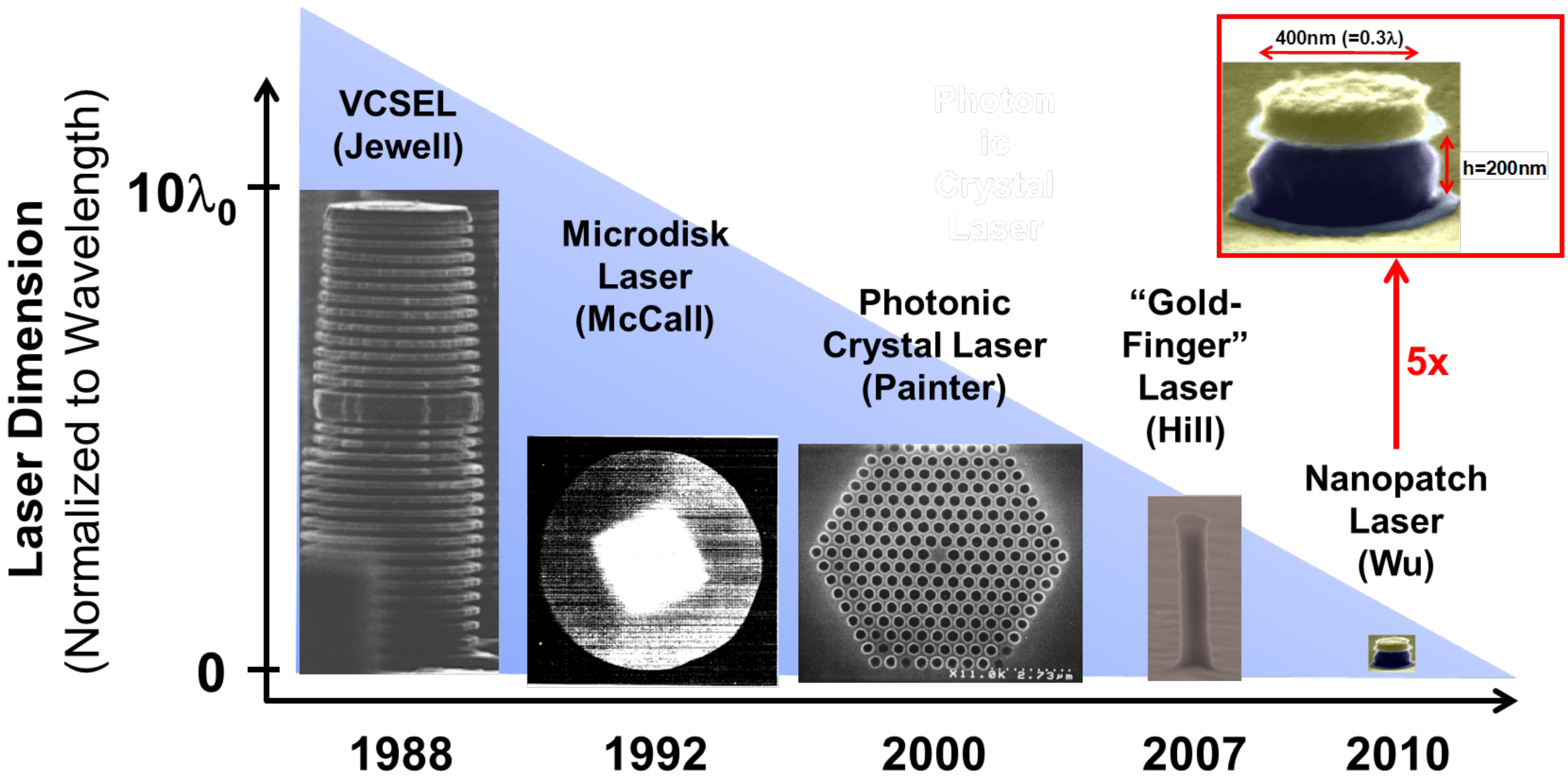
$$g = g_{th} = \frac{\alpha_i}{\Gamma} + \frac{1}{2\Gamma L} \ln\left(\frac{1}{R_1 R_2}\right) = \frac{\alpha_i + \alpha_m}{\Gamma}$$

α_i : intrinsic loss

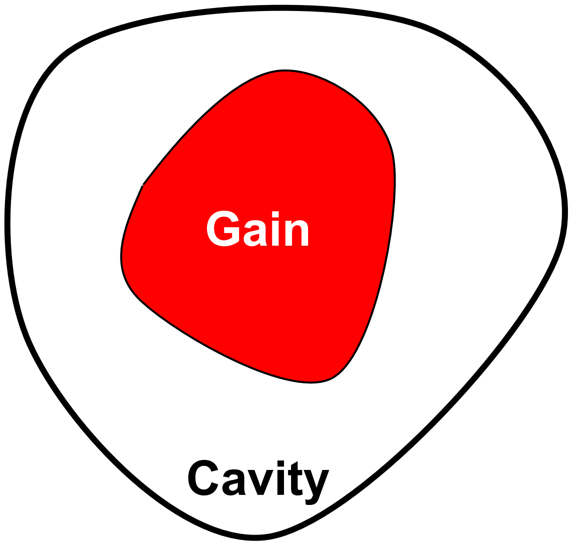
$\alpha_m = \dfrac{1}{2L} \ln\left(\dfrac{1}{R_1 R_2}\right)$: mirror loss (i.e., output light)

Modern Lasers

- **Optical cavity does not necessarily consist of mirrors**



Generic Description of Optical Cavity



Quality Factor:

$$Q = \frac{\text{Energy Stored}}{\text{Energy Dissipated per Cycle}}$$

$$Q = \frac{\omega}{\Delta\omega}$$

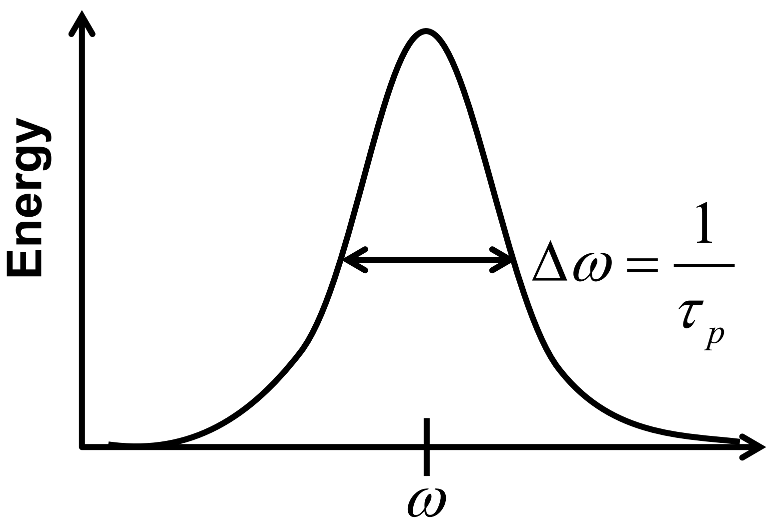
$$\Delta\omega = \frac{1}{\tau_p}$$

τ_p : photon lifetime [sec]

$$\frac{1}{\tau_p} = \alpha \frac{c}{n}$$

$\left(\begin{array}{c} \alpha\text{: loss rate per cm} \\ 1/\tau_p\text{: loss rate per sec} \end{array} \right)$

$$\boxed{Q = \omega \tau_p}$$



Photon Lifetime and Spectral Width

Decay of optical energy when input is turned off (ring-down measurement):

$$I(t) = I_0 e^{-t/\tau_p} \quad \text{for } t \geq 0$$

Electrical (optical) field:

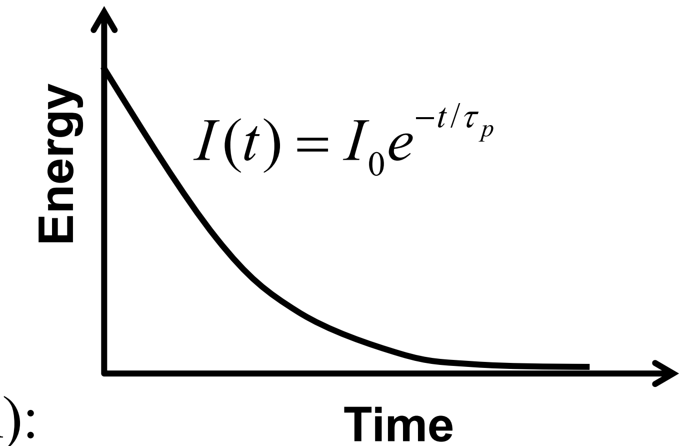
$$E(t) = E_0 e^{j\omega_0 t} e^{-t/2\tau_p} \quad \text{for } t \geq 0$$

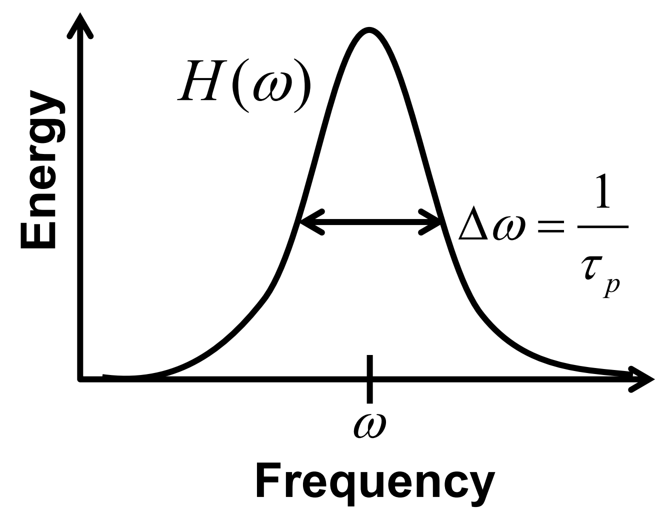
Frequency domain response (Fourier transform):

$$H(\omega) = \int_0^\infty e^{j\omega_0 t} e^{-t/2\tau_p} e^{-j\omega t} dt = \frac{1}{j(\omega - \omega_0) + 1/2\tau_p}$$

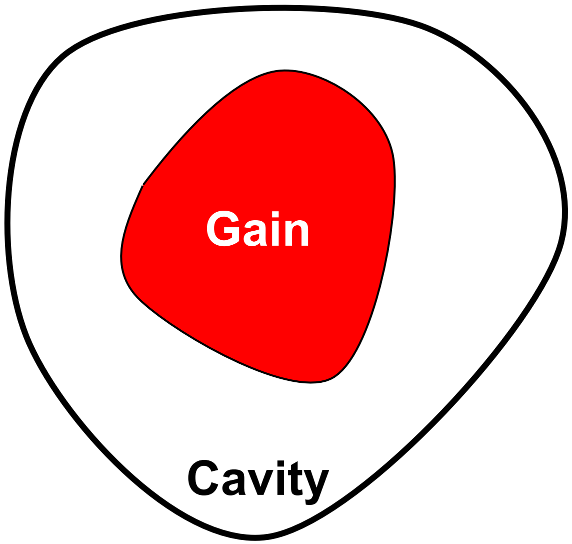
FWHM of $|H(\omega)|^2$: $\quad \omega - \omega_0 = \pm \dfrac{1}{2\tau_p}$

$$\boxed{\Delta\omega = \frac{1}{\tau_p}}$$





Threshold Condition of Generic Lasers

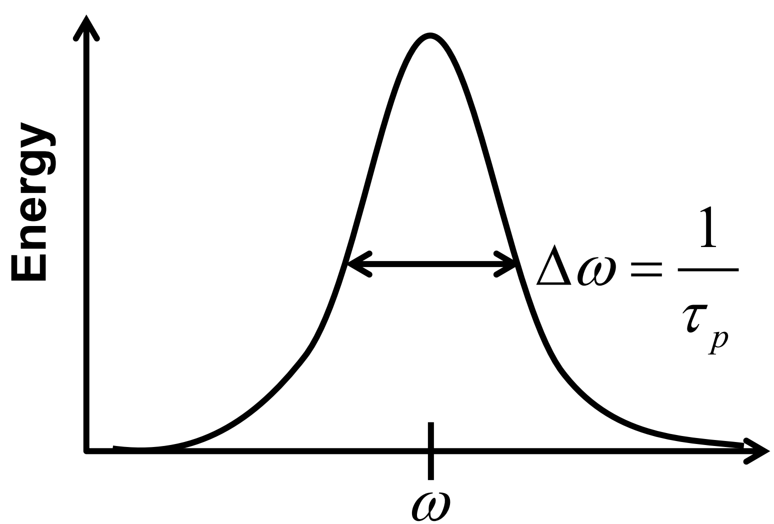


Gain = Loss

(rate of gain = rate of loss)

$$\Gamma g_{th} \frac{c}{n} = \frac{1}{\tau_p} = \frac{\omega}{Q}$$

$$\boxed{g_{th} = \frac{\omega}{Q} \frac{n}{\Gamma c}}$$



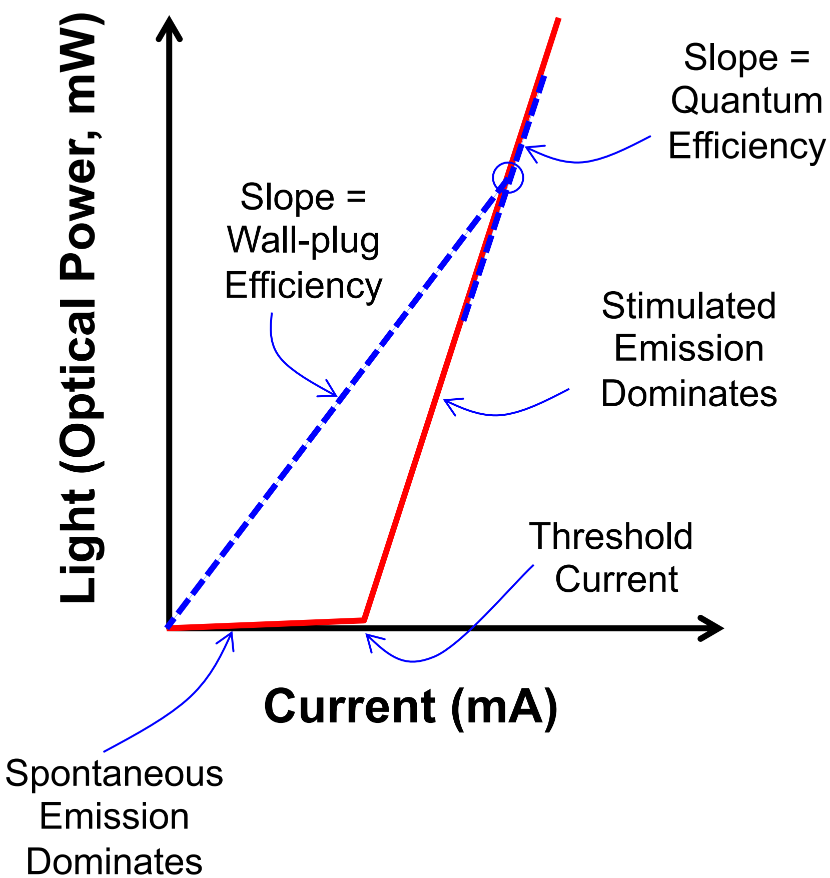
Quantum efficiency:

$$\eta = \frac{\alpha_m}{\alpha_m + \alpha_i} = \frac{Q_{rad}^{-1}}{Q_{rad}^{-1} + Q_{loss}^{-1}} = \frac{Q_{rad}^{-1}}{Q^{-1}}$$

$$\boxed{\eta = \frac{Q}{Q_{rad}}}$$

Prof. Ming Wu

L-I Curve of Semiconductor Lasers



- Distinctive threshold (at least in classical lasers)
- Semiconductor laser is a forward-biased p-n junction, so mainly a current-biased device
- Threshold current :
  - Minimum current at which the laser starts to “lase”
- Quantum efficiency
  - “Differential” electrical-to-optical conversion efficiency, i.e., how many photons generated by injected electrons beyond threshold
- Wall-plug efficiency
  - Total electrical-to-optical conversion efficiency

Typical Q of Semiconductor Laser

Edge-emitting laser:

$L = 100\mu m, \quad R = 30\%, \quad \omega \sim 100 THz \; , \tau_p \sim 1 ps, \quad Q \sim 600$

Vertical Cavity Surface-Emitting Laser (VCSEL)

$L = 1\mu m, R = 99\%, \quad Q \sim 700$

Microdisk (Whispering Gallery Mode or WGM) Laser

$Q \sim 1000$ (up to 10^{11} possible in low loss materials)

Photonic crystal laser: $Q \sim 1000$ (up to 10^{6} possible)

Metal cavity laser (plasmonic laser): $Q \sim 10$ to 100

Gain Cross-Section

Gain cross-section (instead of gain coefficient) is often used to measure the gain in gas or solid-state lasers:

$\sigma : \ [\text{cm}^2]$

Gain cross-section is related to gain by:

$g = N\sigma$

where N is concentration of active molecules

For comparison, in semiconductor lasers:

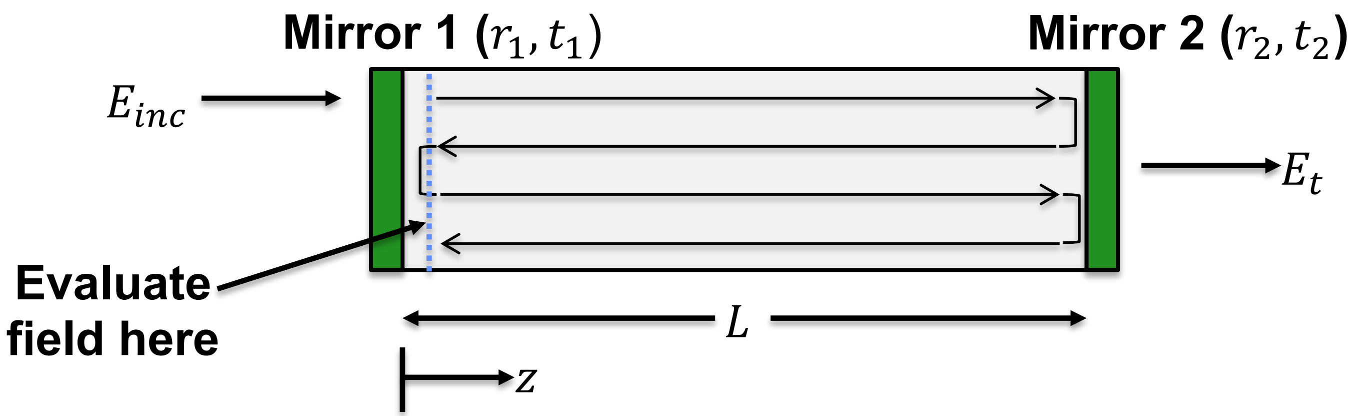
$g \sim 100 \ \text{cm}^{-1}$

$N \sim 10^{18} \ \text{cm}^{-3}$ (typical electron concentration at threshold)

$\sigma \sim 10^{-16} \ \text{cm}^2 \ (= (0.1\text{nm})^2 \ )$

Note: more precise relation between gain and carrier concentration will be discussed in future lectures

Fabry-Perot cavity modes



Field just to the right of Mirror 1 and propagating in +z direction:

$$E^+ = E_{inc} t_1 (1 + r_1 r_2 e^{-jk2L} + r_1 r_2 r_1 r_2 e^{-jk4L} + (r_1 r_2 e^{-jk2L})^3 + \ldots)$$

$$\sum_k^\infty a r^k = \frac{a}{1-r}$$

$$E^+ = E_{inc} t_1 \frac{1}{1 - r_1 r_2 e^{-j2\theta}} \quad \text{where } \theta = kL$$

E^+ **is the field traveling to the right (+z direction)**

Fabry-Perot cavity modes

$$\text{Transmission } T = \frac{I_t}{I_{inc}} = \frac{\langle S_t \rangle}{\langle S_{inc} \rangle} = \frac{\dfrac{|E^+|^2 t_2^2}{2\eta_0}}{\dfrac{|E_{inc}|^2}{2\eta_0}}$$

Assume that $r_{1,2}$ and $t_{1,2}$ are real for simplicity

$$= \frac{\left| E_{inc} t_1 \dfrac{1}{1 - r_1 r_2 e^{-j2\theta}} \right|^2 t_2^2}{|E_{inc}|^2}$$

$$= \frac{t_1^2 t_2^2}{\left|1 - r_1 r_2 e^{-j2\theta}\right|^2}$$

$$= \frac{(1 - r_1^2)\eta_{cavity}\eta_0^{-1}(1 - r_2^2)\eta_0\eta_{cavity}^{-1}}{\left|1 - r_1 r_2 e^{-j2\theta}\right|^2}$$

$$= \frac{(1 - r_1^2)(1 - r_2^2)}{\left|1 - r_1 r_2 e^{-j2\theta}\right|^2}$$

$\langle S \rangle$: Time-average Poynting vector (intensity)

η_0 : Impedance of medium outside cavity

η_{cavity} : Impedance of medium inside cavity

t_1: transmission of mirror 1

r_1: reflectivity of mirror 1

t_2: transmission of mirror 2

r_2: reflectivity of mirror 2

$$t_1^2 \eta_0 \eta_{cavity}^{-1} + r_1^2 = 1$$

$$t_2^2 \eta_0^{-1} \eta_{cavity} + r_2^2 = 1$$

Fabry-Perot cavity modes

$$= \frac{(1-r_1^2)(1-r_2^2)}{\left|1-r_1 r_2 e^{-j2\theta}\right|^2}$$

$$= \frac{(1-r_1^2)(1-r_2^2)}{(1-r_1 r_2 e^{j2\theta})(1-r_1 r_2 e^{-j2\theta})}$$

$$= \frac{(1-r_1^2)(1-r_2^2)}{1-r_1 r_2\left[e^{j2\theta}+e^{-j2\theta}\right]+(r_1 r_2)^2}$$

$$= \frac{(1-r_1^2)(1-r_2^2)}{1-2r_1 r_2\left[\cos 2\theta\right]+(r_1 r_2)^2}$$

$$= \frac{(1-r_1^2)(1-r_2^2)}{1-2r_1 r_2\left[1-2\sin^2\theta\right]+(r_1 r_2)^2}$$

$$= \frac{(1-r_1^2)(1-r_2^2)}{1-2r_1 r_2+4r_1 r_2\sin^2\theta+(r_1 r_2)^2}$$

$$\boxed{T = \frac{(1-r_1^2)(1-r_2^2)}{(1-r_1 r_2)^2+4r_1 r_2\sin^2 kL}}$$

$\langle S \rangle$: Time-average Poynting vector (intensity)

η_0 : Impedance of medium outside cavity

η_{cavity} : Impedance of medium inside cavity

t_1: transmission of mirror 1

r_1: reflectivity of mirror 1

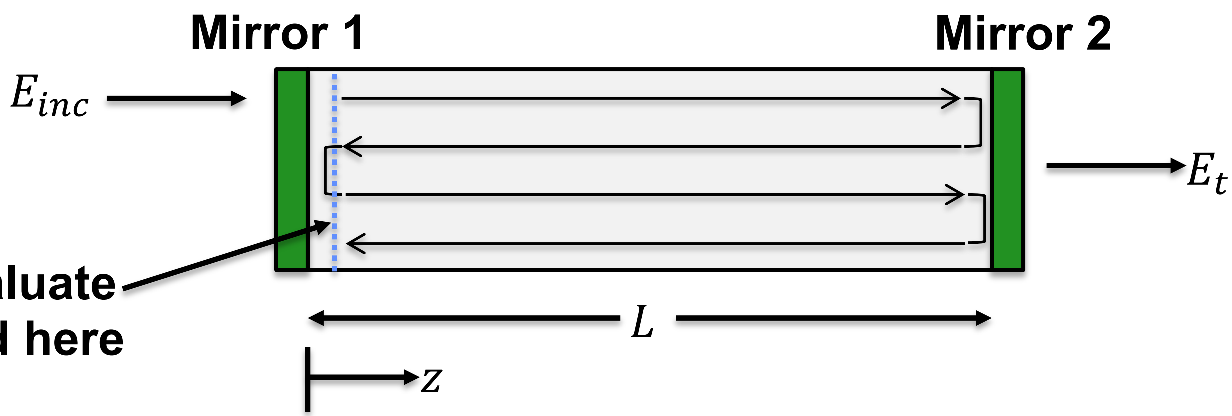
t_2: transmission of mirror 2

r_2: reflectivity of mirror 2

$$t_1^2\eta_0\eta_{cavity}^{-1}+r_1^2=1$$

$$t_2^2\eta_0^{-1}\eta_{cavity}+r_2^2=1$$

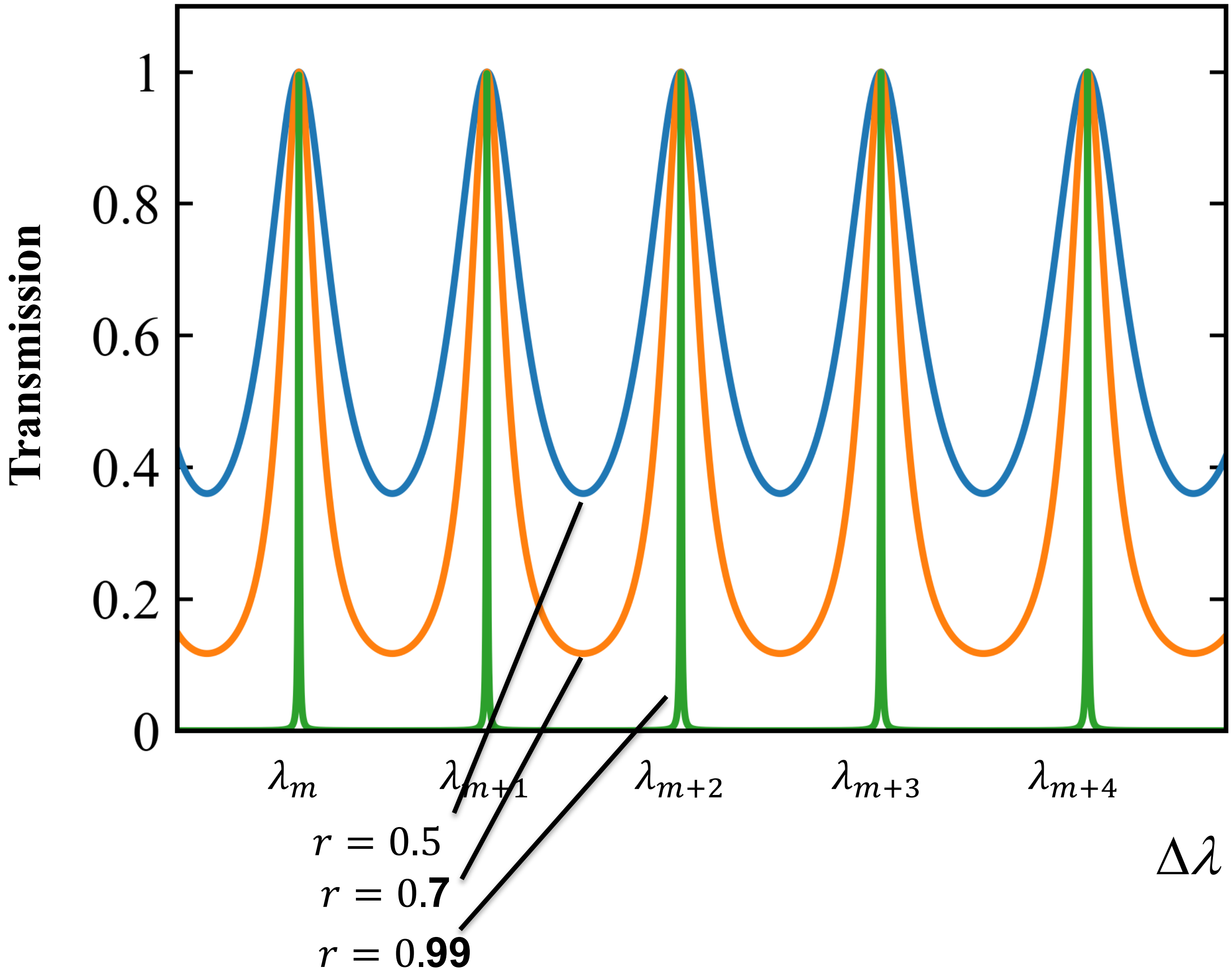
Fabry-Perot cavity modes



Resonance condition: $\sin^2\theta = 0 \rightarrow kL = m\pi$

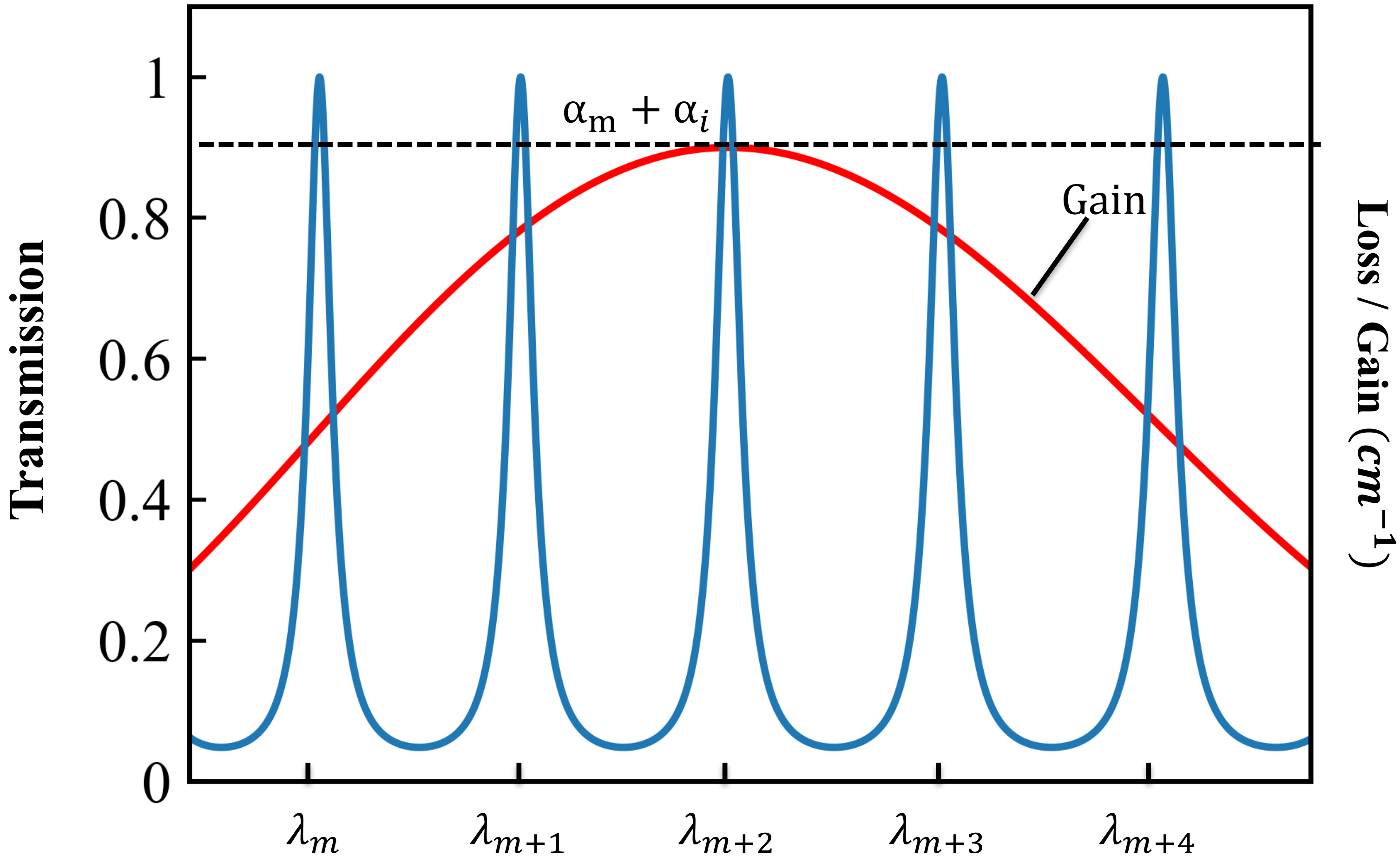
$$L = \frac{m\lambda_0}{2n} \quad \text{m = integer}$$

Fabry-Perot cavity modes



$$\Delta\lambda \cong \frac{\lambda^2}{2nL}$$

Fabry-Perot cavity modes



e.g. with simple gain curve
Threshold is achieved for one Fabry-Perot mode.