



EE 232 Lightwave Devices

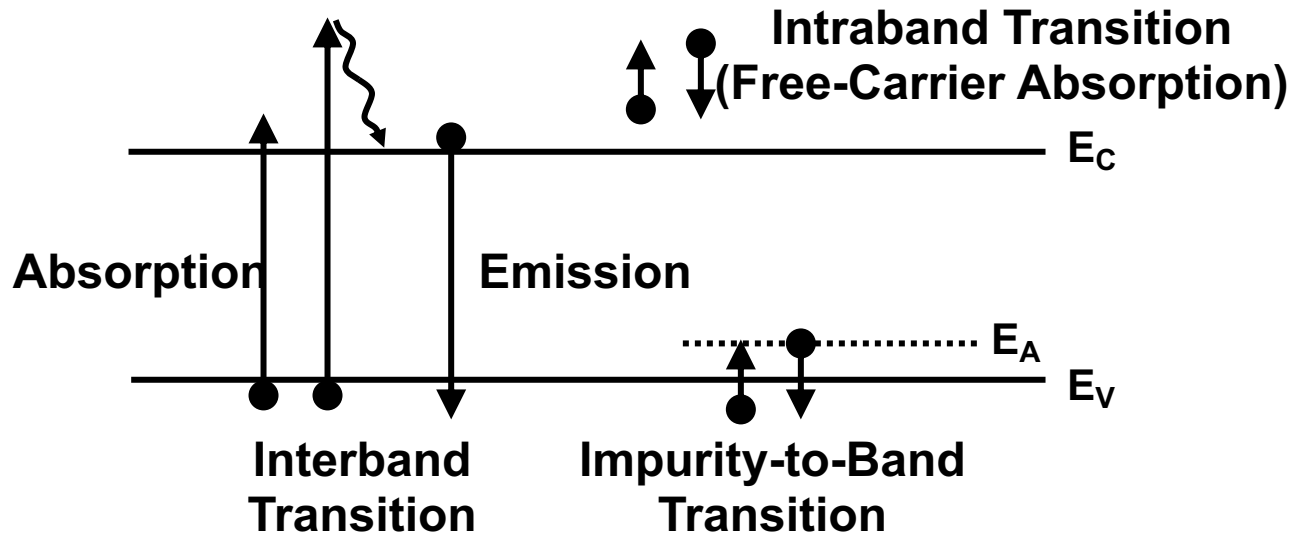
Lecture 3: Basic Semiconductor Physics and Optical Processes

Instructor: Ming C. Wu

University of California, Berkeley
Electrical Engineering and Computer Sciences Dept.



Optical Properties of Semiconductors



- Optical transitions

- Absorption: exciting an electron to a higher energy level by absorbing a photon
- Emission: electron relaxing to a lower energy state by emitting a photon



Band-to-Band Transition

- Since most electrons and holes are near the band-edges, the photon energy of band-to-band (or interband) transition is approximately equal to the bandgap energy:

$$h\nu = E_g$$

- The optical wavelength of band-to-band transition can be approximated by

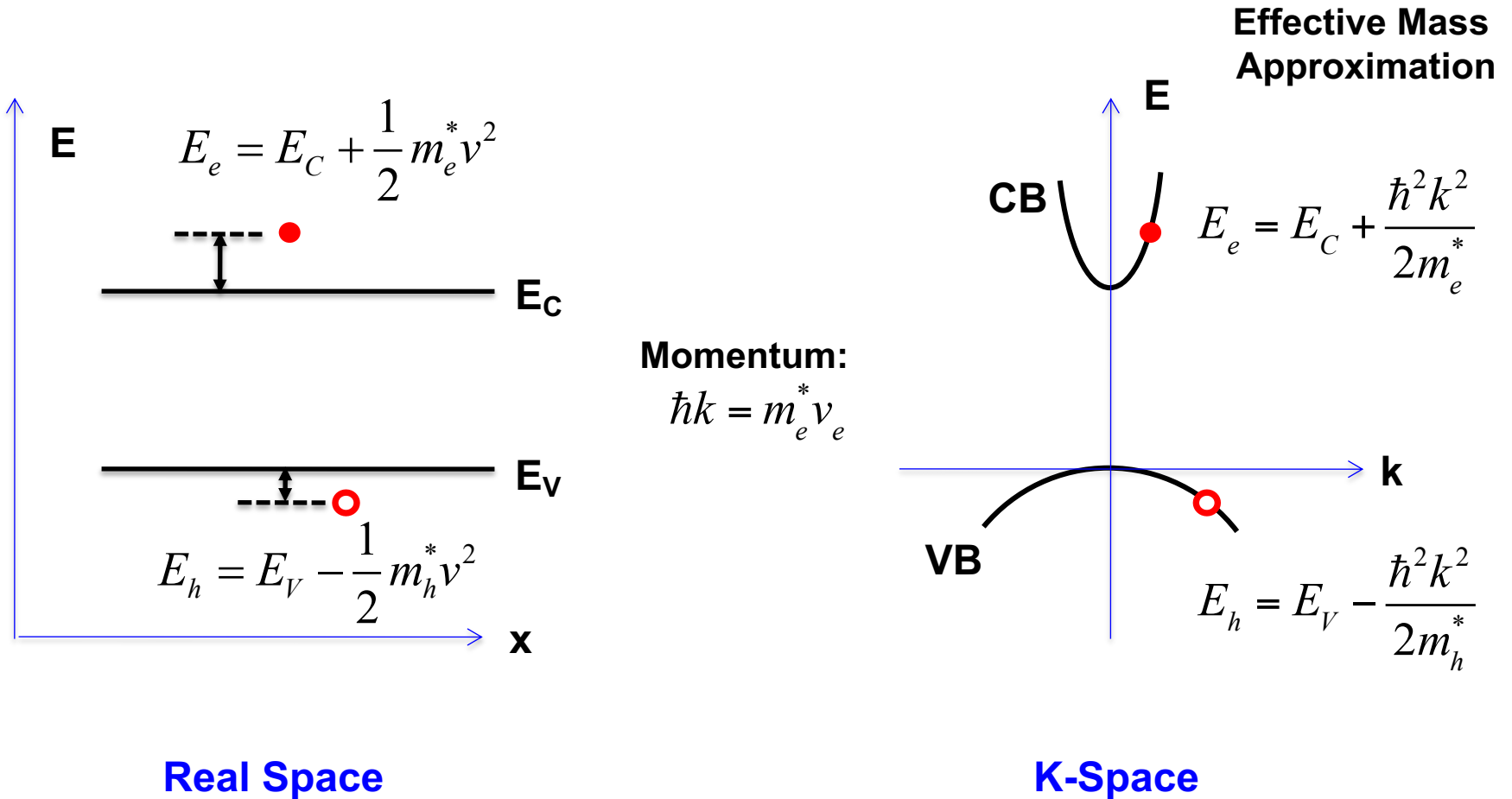
$$\lambda = \frac{c}{\nu} = \frac{hc}{E_g} \approx \frac{1.24}{E_g}$$

λ : wavelength in μm

E_g : energy bandgap in eV

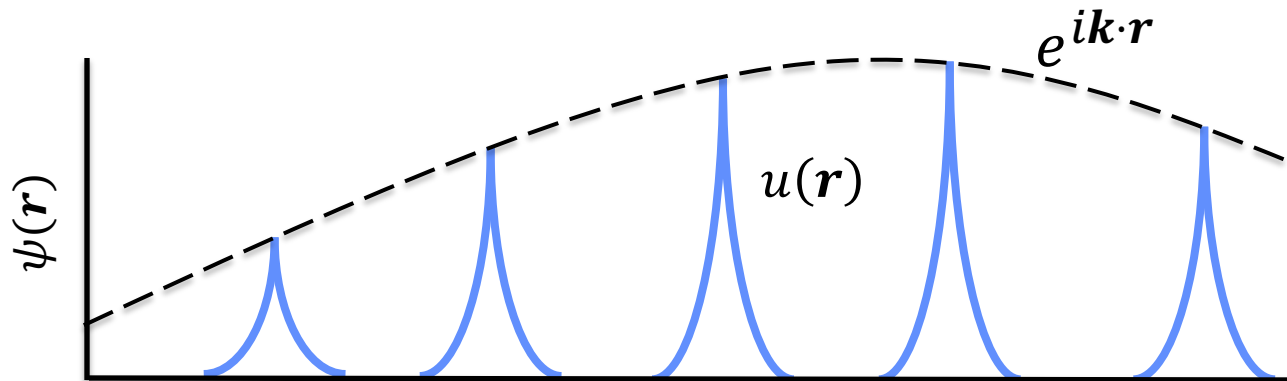
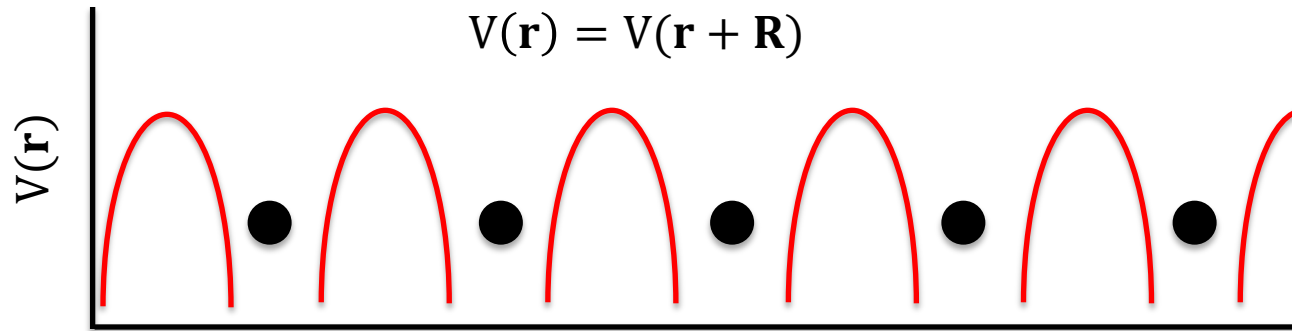


Energy Band Diagram in Real Space and k-Space





Bloch Theorem

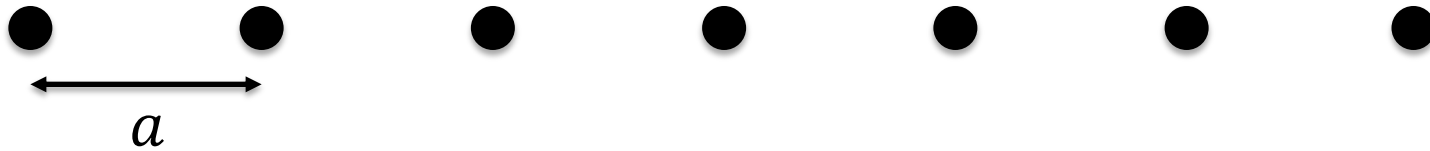


$$\psi(\mathbf{r}) = u(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}} \quad (\text{Bloch Theorem})$$



Bloch Theorem (1D proof)

Linear chain of N periodic atoms



$$|\psi(x+a)|^2 = |\psi(x)|^2$$

$$\psi(x+a) = C\psi(x), \quad |C| = 1$$

Periodic Boundary Condition:

$$\psi(x+Na) = \psi(x)$$

$$\psi(x+Na) = C^N \psi(x) = \psi(x)$$

$$C^N = 1 \Rightarrow C = e^{\frac{i2\pi s}{Na}}, \quad s = 1..N-1$$



Bloch Theorem (1D proof)

$$\psi(x+a) = C\psi(x) = e^{\frac{i2\pi s}{Na}} \psi(x) = e^{ika} \psi(x)$$

$$\text{where } k = \frac{2\pi s}{Na}$$

Multiply both sides of the above equation by $e^{-ik(x+a)}$:

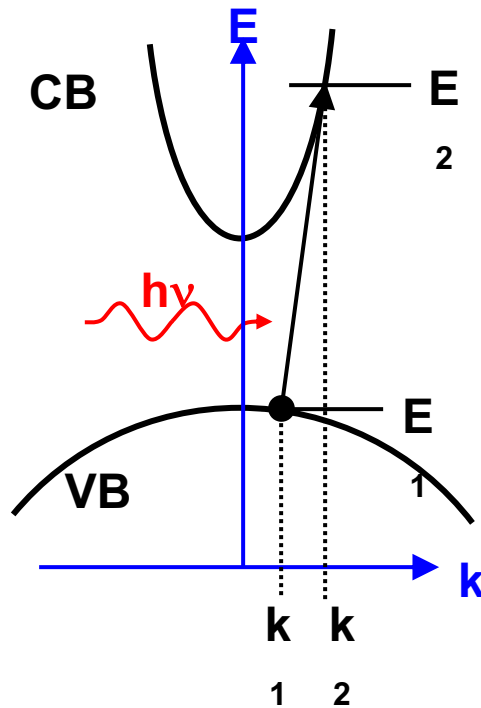
$$e^{-ik(x+a)} \psi(x+a) = e^{-ik(x+a)} e^{ika} \psi(x) = e^{-ikx} \psi(x)$$

Define $u(x) \equiv e^{-ikx} \psi(x) \rightarrow u(x+a) = u(x)$

$$\Rightarrow \psi(x) = e^{ikx} u(x)$$



Conservation of Energy and Momentum



Optical transitions are “vertical” lines

- Conditions for optical absorption and emission:
 - Conservation of energy

$$E_2 - E_1 = h\nu$$

- Conservation of momentum

$$k_2 - k_1 = k_{hv}$$

$$k_2, k_1 \sim \frac{2\pi}{a}$$

$$k_{hv} \sim \frac{2\pi}{\lambda}$$

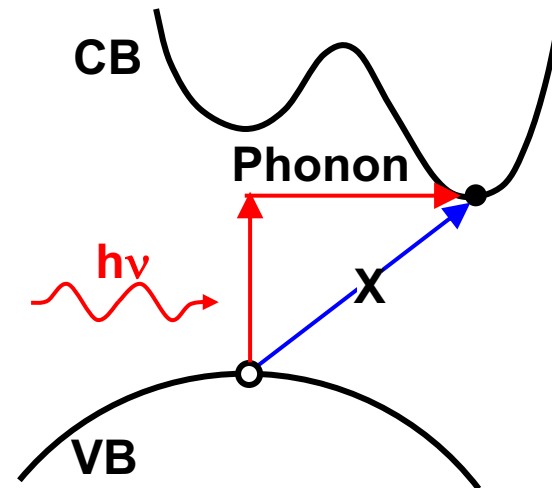
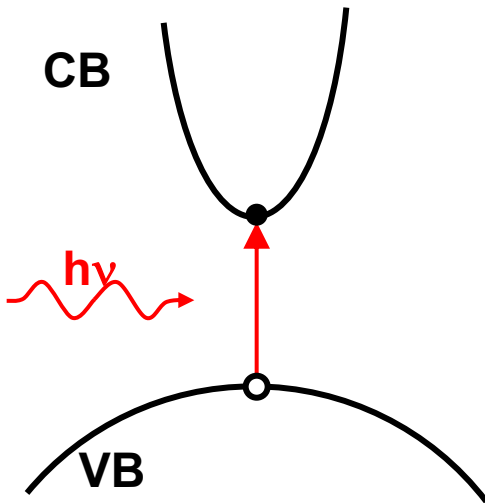
$$(a \sim 0.5\text{nm}) \ll (\lambda \sim 1\mu\text{m})$$

Lattice Constant

$$\Rightarrow k_2 = k_1$$



Direct vs Indirect Bandgaps

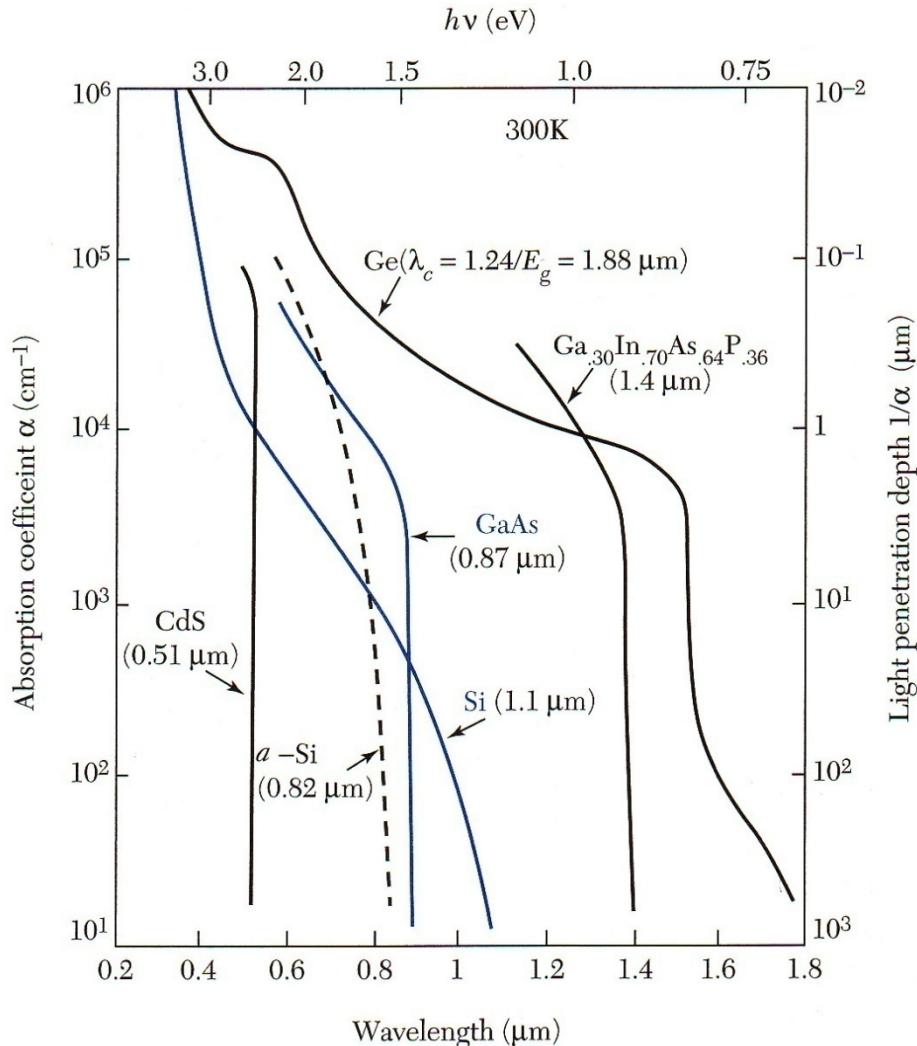


- Direct bandgap materials
 - CB minimum and VB maximum occur at the same k
 - Examples
 - GaAs, InP, InGaAsP
 - $(\text{Al}_x\text{Ga}_{1-x})\text{As}$, $x < 0.45$

- Indirect bandgap materials
 - CB minimum and VB maximum occur at different k
 - Example
 - Si, Ge
 - $(\text{Al}_x\text{Ga}_{1-x})\text{As}$, $x > 0.45$
 - Not “optically active”



Absorption Coefficient



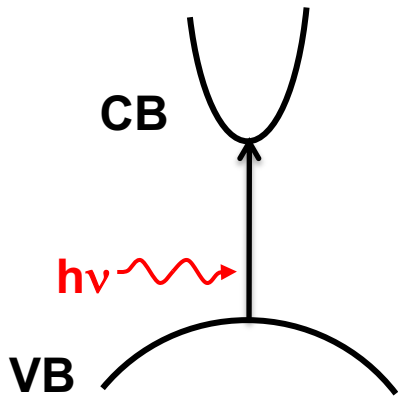
- Light intensity decays exponentially in semiconductor:

$$I(x) = I_0 e^{-\alpha x}$$

- Direct bandgap semiconductor has a sharp absorption edge
- Si absorbs photons with $h\nu > E_g = 1.1$ eV, but the absorption coefficient is small
 - Sufficient for CCD
- At higher energy (~ 3 eV), absorption coefficient of Si becomes large again, due to direct bandgap transition to higher CB

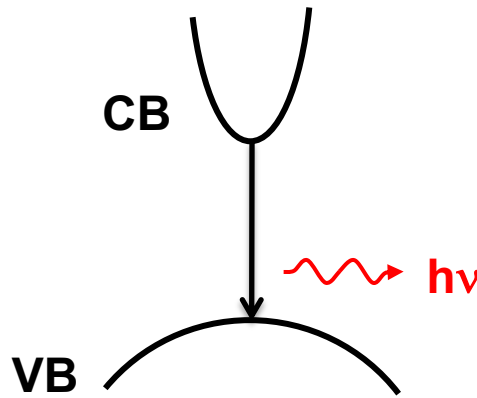


Band-to-Band Transition



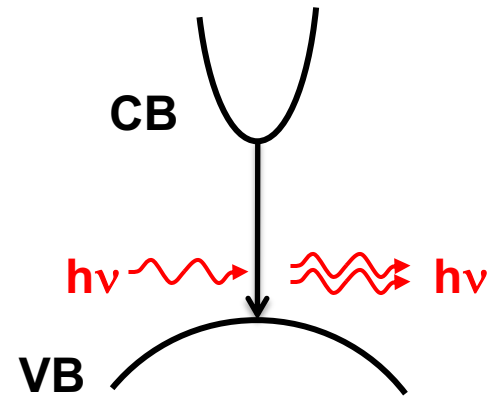
Absorption

Photodetectors;
Solar Cells



Spontaneous
Emission

LED



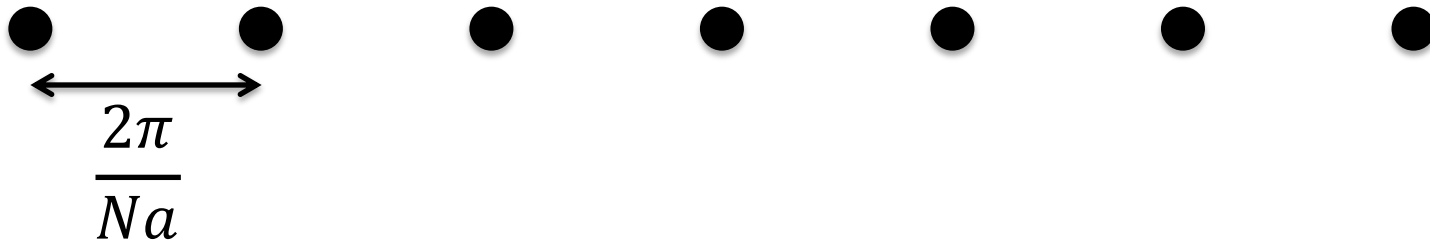
Stimulated
Emission

Optical Amplifiers;
Semiconductor Lasers



Density of states in k-space (1D)

Linear chain of N periodic atoms in k-space



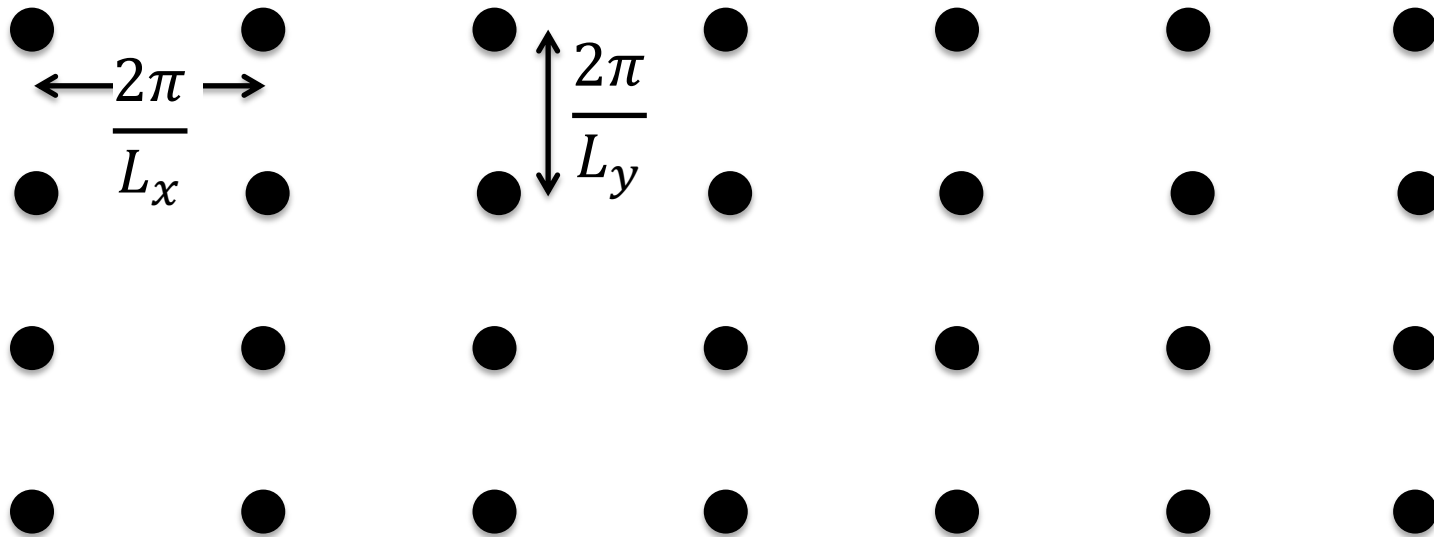
Number of states per unit length:

$$N = \frac{2}{L_x} \sum_{k_x} \rightarrow \frac{2}{L_x} \int \frac{1}{\left(\frac{2\pi}{Na}\right)} dk = \frac{2}{L_x} \int \frac{1}{\left(\frac{2\pi}{L_x}\right)} dk = \frac{2}{L_x} \int \frac{L_x}{2\pi} dk = 2 \int \frac{1}{2\pi} dk$$

(Factor of two from spin degeneracy)



Density of states in k-space (2D and 3D)



Number of states per unit area (2D crystal):

$$N = \frac{2}{L_x L_y} \sum_{k_x} \sum_{k_y} \rightarrow \frac{2}{L_x L_y} \int \frac{L_x L_y}{(2\pi)^2} d^2 \mathbf{k} = \int \frac{2}{(2\pi)^2} d^2 \mathbf{k}$$

Number of states per unit volume (3D crystal):

$$N = \frac{2}{L_x L_y L_z} \sum_{k_x} \sum_{k_y} \sum_{k_z} \rightarrow \frac{2}{L_x L_y L_z} \int \frac{L_x L_y L_z}{(2\pi)^3} d^3 \mathbf{k} = \int \frac{2}{(2\pi)^3} d^3 \mathbf{k}$$



Density of States for CB

- Number of electron states between E and $E + \Delta E$ per unit volume

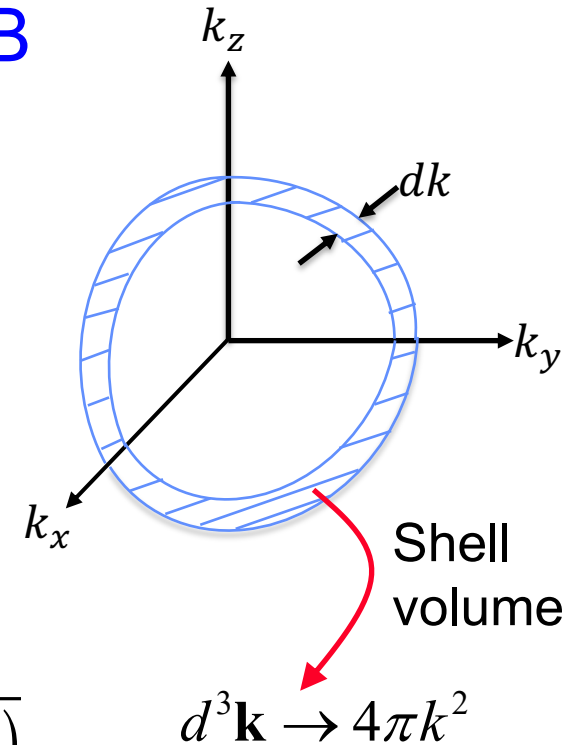
$$N = 2 \int \frac{1}{(2\pi)^3} d^3\mathbf{k} = 2 \int \frac{4\pi k^2}{(2\pi)^3} dk = \int \frac{k^2}{\pi^2} dk = \int \rho_k(k) dk$$

$$\rho_k(k) dk = \frac{k^2}{\pi^2} dk \quad \text{Convert to } \rho_e(E) dE$$

$$E = E_C + \frac{\hbar^2 k^2}{2m_e^*} \Rightarrow dE = \frac{\hbar^2 k}{m_e^*} dk \quad \text{and} \quad k = \frac{\sqrt{2m_e^*(E - E_C)}}{\hbar}$$

$$\frac{k^2}{\pi^2} dk = \frac{k}{\pi^2} k dk = \frac{k}{\pi^2} \left(\frac{m_e^*}{\hbar^2} dE \right) = \frac{m_e^*}{\hbar^2 \pi^2} \frac{\sqrt{2m_e^*(E - E_C)}}{\hbar} dE = \rho_e(E) dE$$

$$\rho_e(E) = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} \sqrt{E - E_C}$$





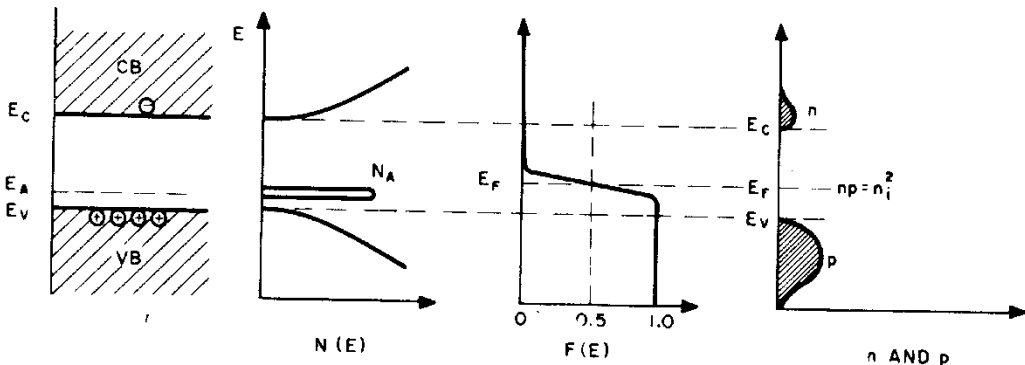
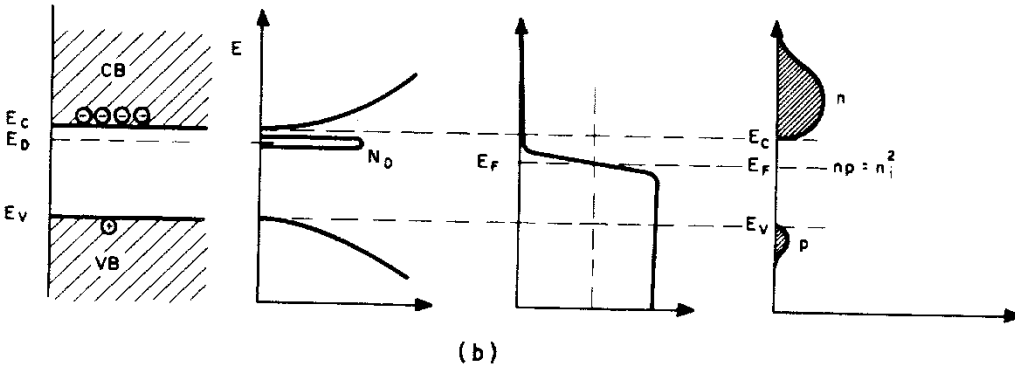
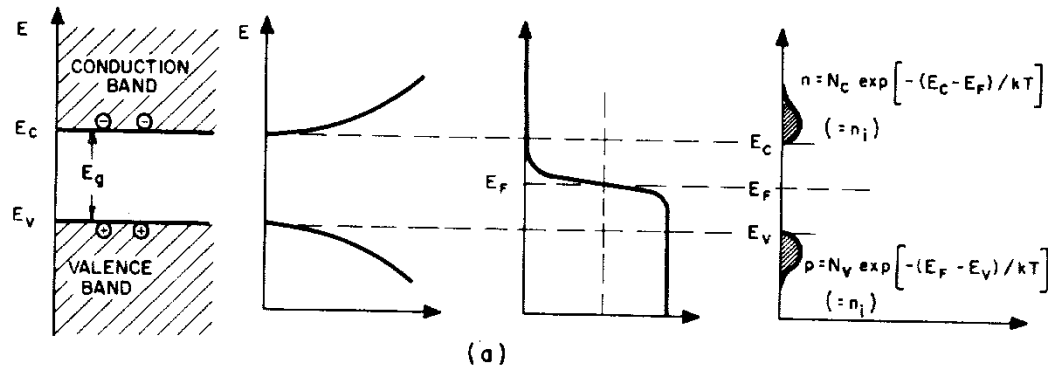
Density of States for VB

- The density of states for VB can be derived in a similar way

$$\rho_h(E) = \frac{1}{2\pi^2} \left(\frac{2m_h^*}{\hbar^2} \right)^{3/2} \sqrt{E_V - E}$$



Review of Semiconductor Physics



Electron and hole concentrations:

$$n = \int_{E_C}^{\infty} f_n(E) \rho_e(E) dE$$

$$p = \int_{-\infty}^{E_V} f_p(E) \rho_h(E) dE$$

Fermi-Dirac distributions:

$$f_n(E) = \frac{1}{1 + \exp\left(\frac{E - F_n}{k_B T}\right)}$$

$$f_p(E) = \frac{1}{1 + \exp\left(\frac{F_p - E}{k_B T}\right)}$$

F_n : electron quasi-Fermi level

F_p : hole quasi-Fermi level



Electron and Hole Concentrations

$$n = \int_{E_C}^{\infty} f_n(E) \rho_e(E) dE = \int_{E_C}^{\infty} \frac{1}{1 + \exp\left(\frac{E - F_n}{k_B T}\right)} \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\left(\frac{h}{2\pi}\right)^2}\right) \sqrt{E - E_C} dE$$

$$n = N_C \cdot F_{1/2}\left(\frac{F_n - E_C}{k_B T}\right)$$

$$N_C = 2 \left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2}$$

$$p = N_V \cdot F_{1/2}\left(\frac{E_V - F_p}{k_B T}\right)$$

$$N_V = 2 \left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2}$$

Fermi-Dirac Integral

$$F_j(\eta) = \frac{1}{\Gamma(j+1)} \int_0^{\infty} \frac{x^j}{1 + e^{x-\eta}} dx$$

Gamma Function

$$\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$$



Approximation of Electron/Hole Concentration

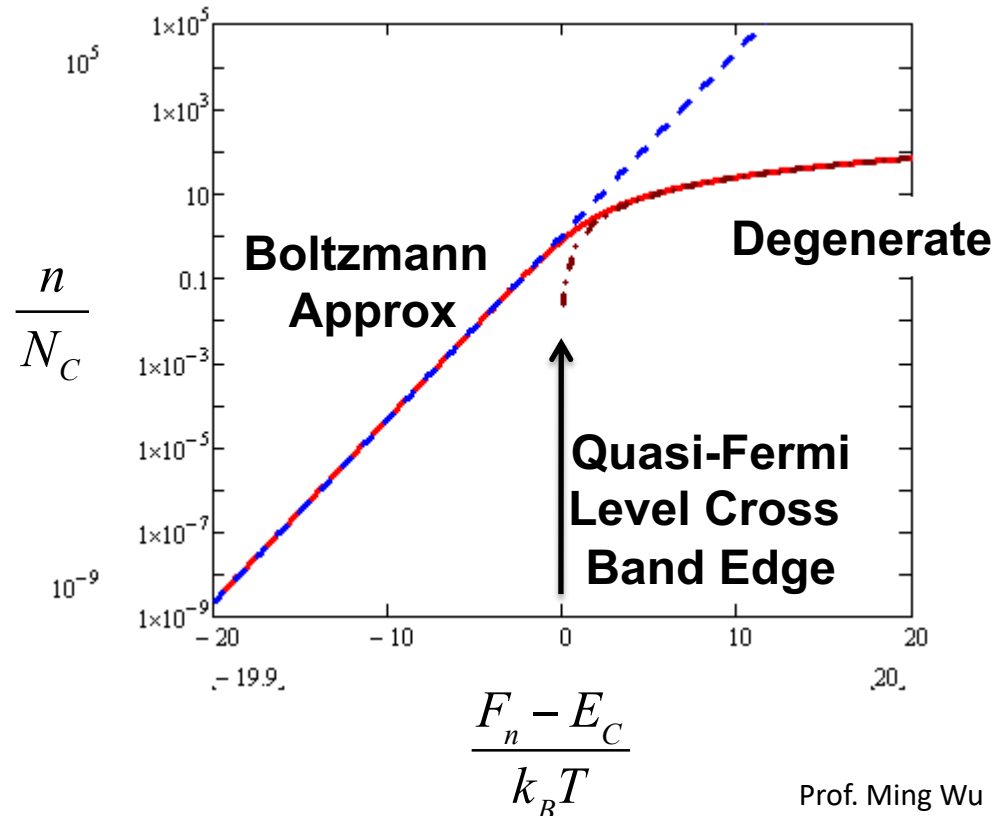
$$F_j(\eta) = \frac{1}{\Gamma(j+1)} \int_0^\infty \frac{x^j}{1+e^{x-\eta}} dx \approx \begin{cases} e^\eta & \text{when } \eta \ll 1 \\ \frac{4}{3} \left(\frac{\eta^3}{\pi} \right)^{1/2} & \text{when } \eta \gg 1 \end{cases}$$

(1) When $F_n \ll E_C$ (Boltzmann approximation)

$$n \approx N_C \cdot e^{-\left(\frac{E_C - F_n}{k_B T}\right)}$$

(2) When $F_n \gg E_C$ (Degenerate)

$$n \approx N_C \cdot \frac{4 \left(\frac{F_n - E_C}{k_B T} \right)^{3/2}}{3\sqrt{\pi}}$$





Typical Values of Density of States

Name		Symbol	Germanium	Silicon	Gallium Arsenide
Energy bandgap at 300 K		E_g (eV)	0.66	1.12	1.424
Effective mass for density of states calculations	Electrons	$m_{e,dos}^*/m_0$	0.56	1.08	0.067
	Holes	$m_{h,dos}^*/m_0$	0.29	0.81	0.47
Effective density of states in the conduction band at 300 K		N_C (cm ⁻³)	1.05×10^{19}	2.82×10^{19}	4.37×10^{17}
Effective density of states in the valence band at 300 K		N_V (cm ⁻³)	3.92×10^{18}	1.83×10^{19}	8.68×10^{18}
Intrinsic carrier density at 300 K		n_i (cm ⁻³)	1.83×10^{13}	8.81×10^9	2.03×10^6
Effective density of states in the conduction band at 100°C (373.15 K)		N_C (cm ⁻³)	1.46×10^{19}	3.91×10^{19}	6.04×10^{17}
Effective density of states in the valence band at 100°C		N_V (cm ⁻³)	5.44×10^{18}	2.54×10^{19}	1.12×10^{19}
Intrinsic carrier density at 100°C		n_i (cm ⁻³)	3.1×10^{14}	8.55×10^{11}	6.24×10^8