EE 232 Lightwave Devices
Lecture 8: Integrated Photonic Components (1)

Instructor: Ming C. Wu

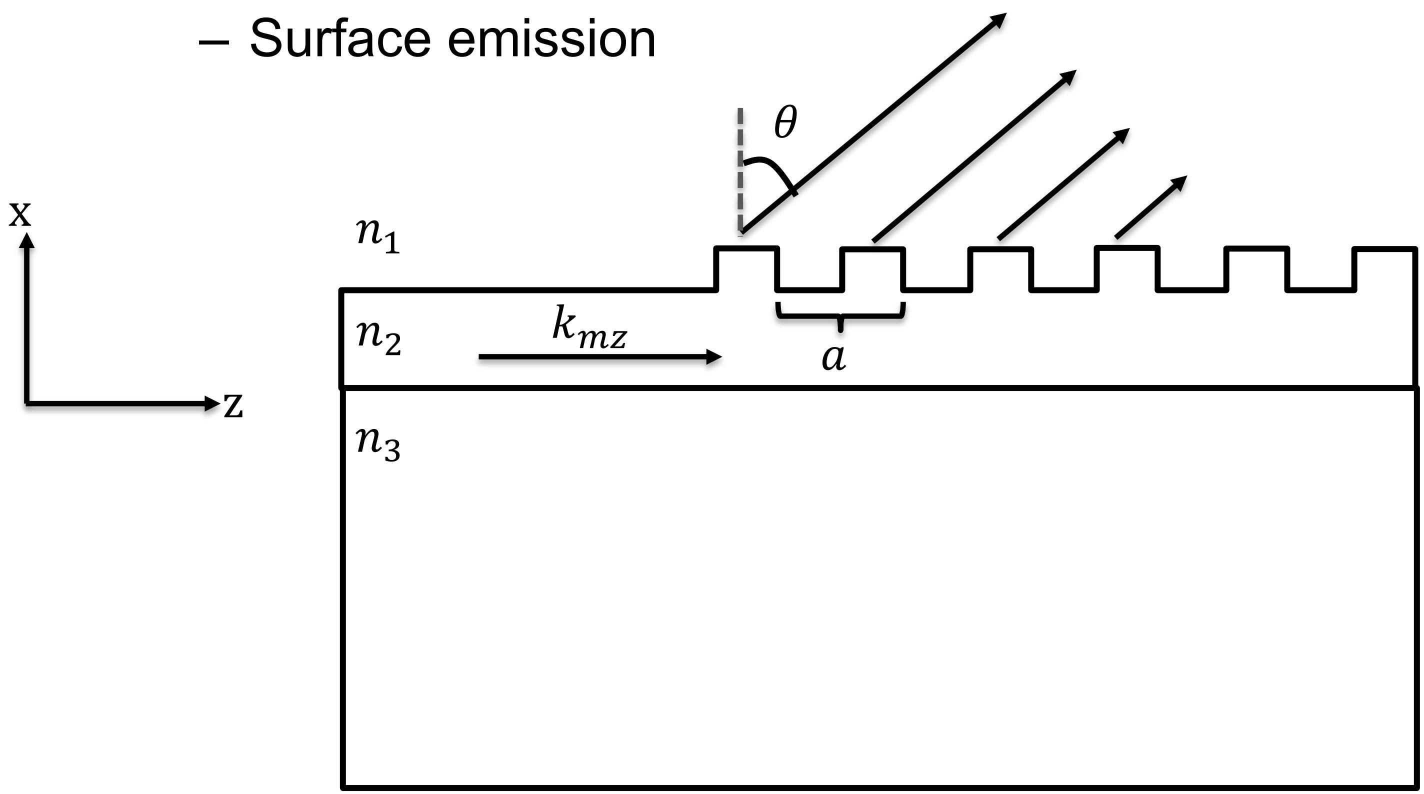
University of California, Berkeley
Electrical Engineering and Computer Sciences Dept.

Reading: Chuang, Chap 8

 Acknowledgment: some lecture materials are provided by Seth Fortuna

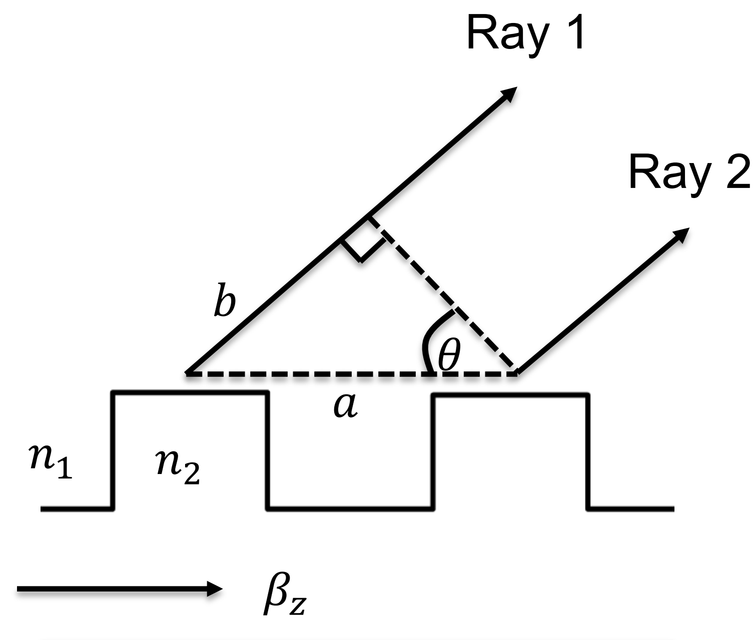
Grating coupler

- Grating couplers are used to couple light in and out of waveguides, for example
  - Couple light to fiber
  - Surface emission



Ray Optics Picture

- Radiation angle determined by constructive interference between consecutive grating teeth



Phase of Ray 2   Phase of Ray 1

$$\beta_z a - k_1 b = 2\pi m \qquad m = 0, \pm 1, \pm 2 \ldots$$

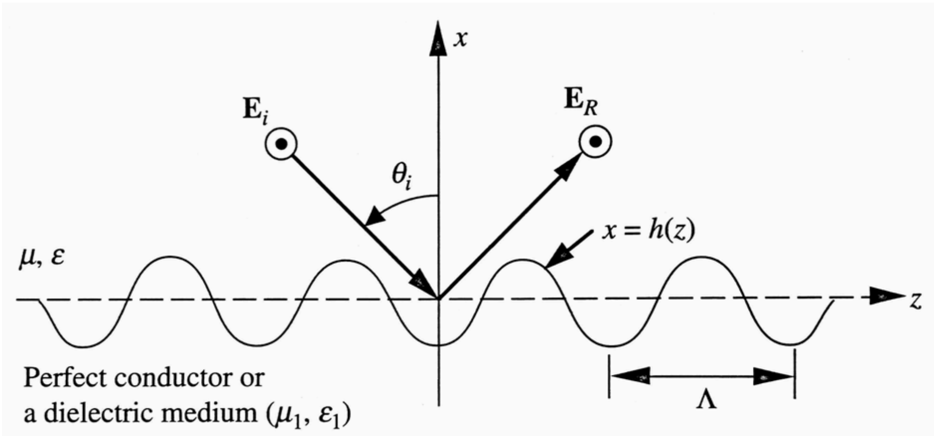
$$\frac{2\pi}{\lambda} n_{eff} a - \frac{2\pi}{\lambda} n_1 b = 2\pi m$$

$$n_{eff} a - \mathrm{n}_1 a \sin(\theta) = m\lambda$$

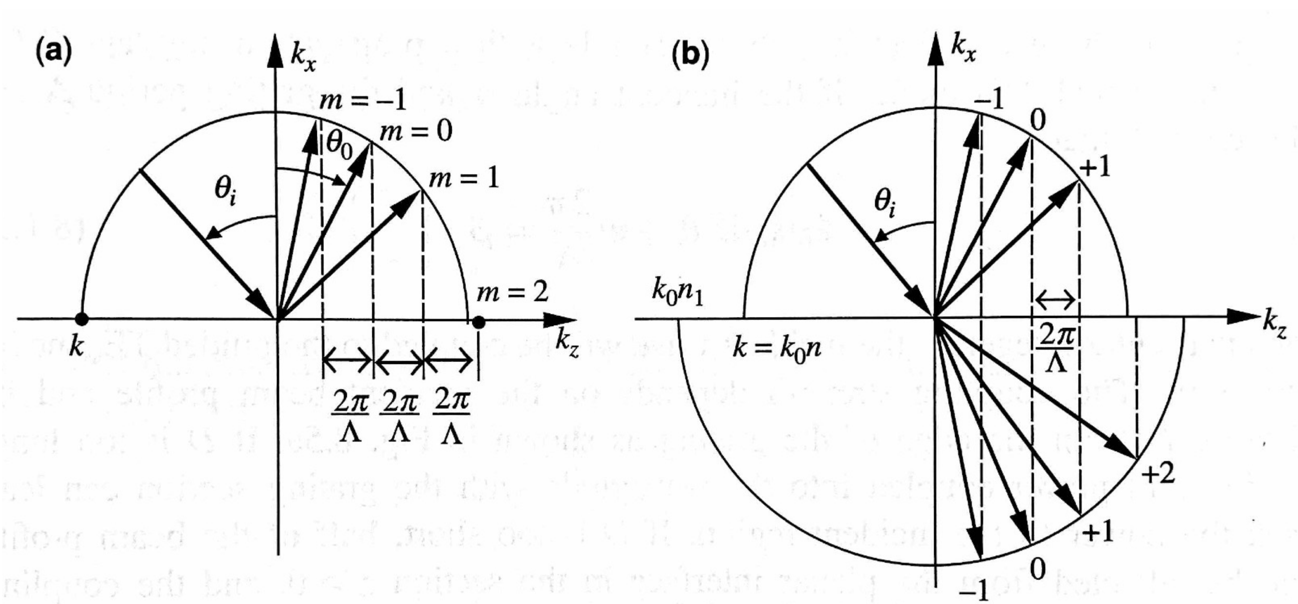
$$\sin(\theta) = \frac{n_{eff} a - m\lambda}{n_1 a}$$

$$\boxed{\sin(\theta) = \frac{n_{eff} - \frac{m\lambda}{a}}{n_1}}$$

Phase Matching Condition for Free Space Grating

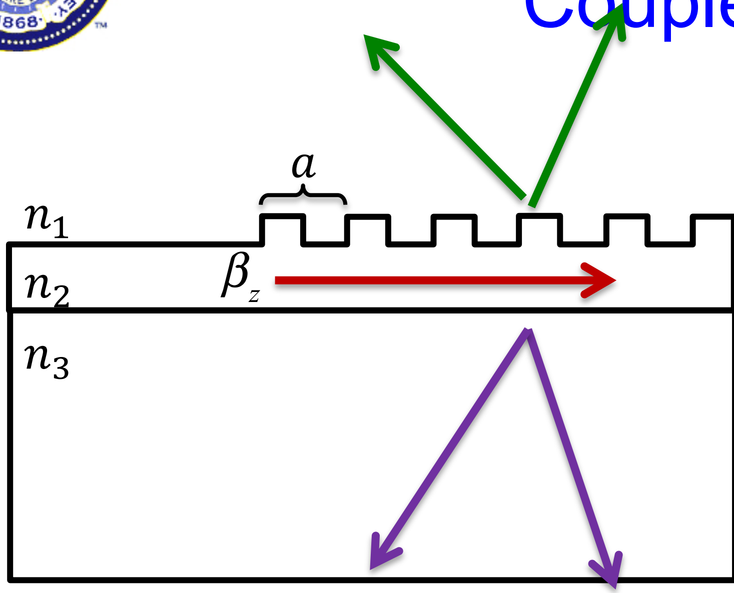


- Grating changes horizontal wave vector (k) by $m\frac{2\pi}{\Lambda}$
  - Spatial Fourier components of the grating
- Momentum (wave vector) is conserved along the grating interface



Reflection Grating Reflection/Transmission Grating

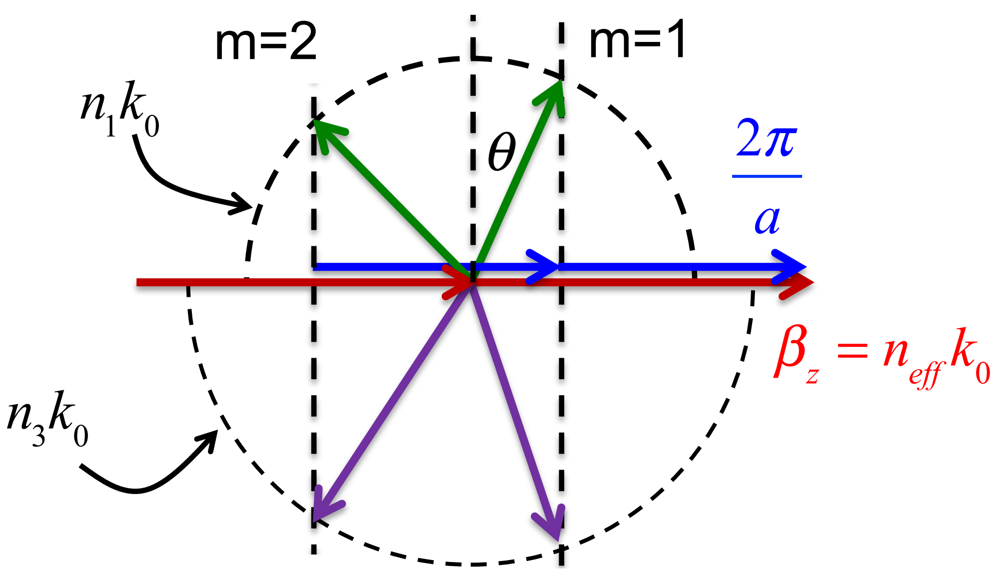
Phase Matching Condition for Grating Coupler in Waveguide



The propagation constant along the z direction is conserved. This is called the ”phase matching condition”:

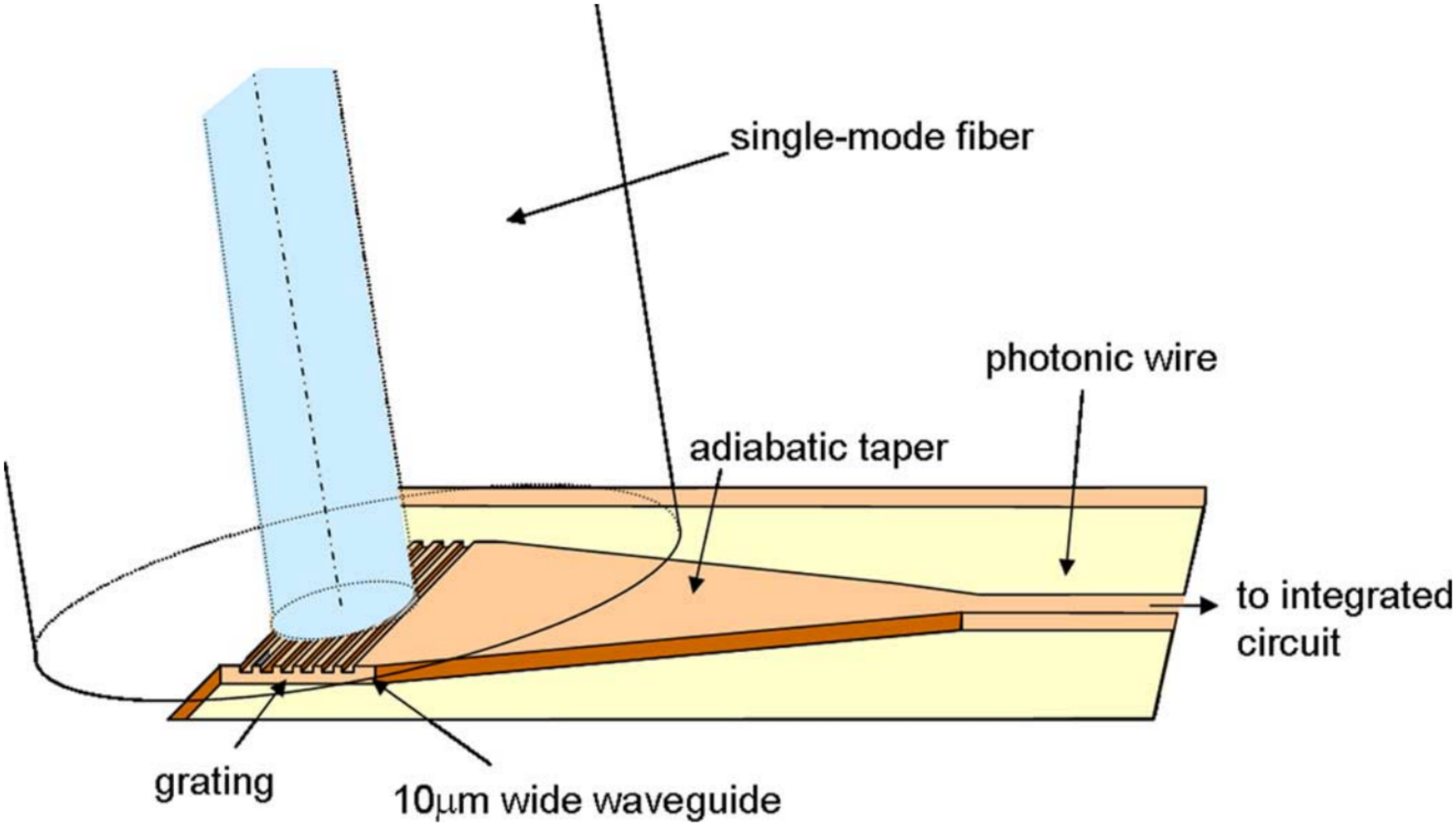
$$k_0 n_1 \sin(\theta) + m\frac{2\pi}{a} = \beta_z = n_{eff} k_0$$

$$\sin(\theta) = \frac{n_{eff} + m\dfrac{\lambda}{a}}{n_1}$$



In most applications, we only want the top emission. The bottom emission can be suppressed through multilayer interference.

Grating coupler to couple light from Si Photonic waveguide to fiber



JOURNAL OF LIGHTWAVE TECHNOLOGY, VOL. 25, NO. 1, JANUARY 2007

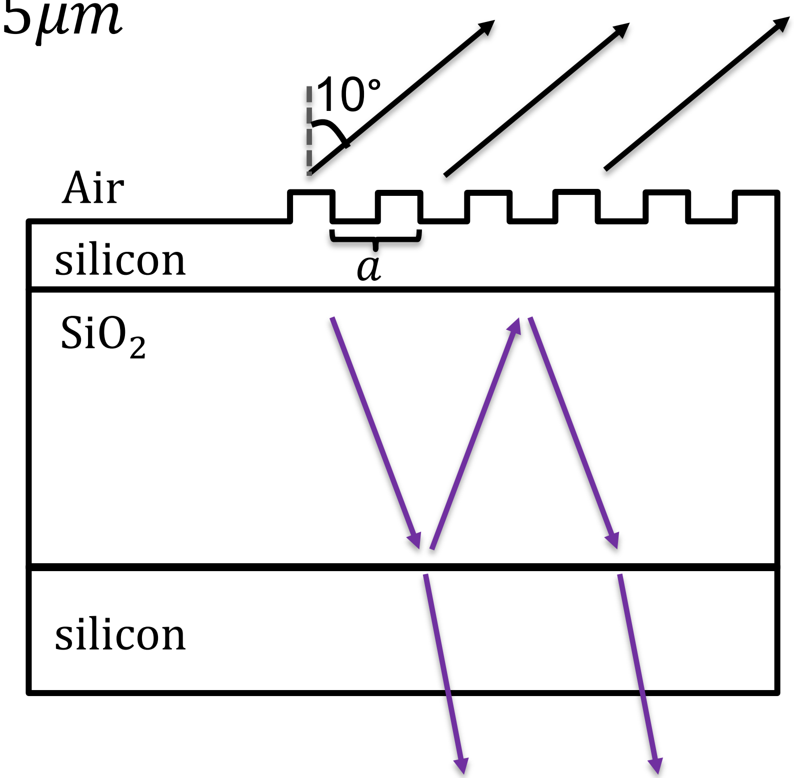
Design example

- Design the grating period such that light is diffracted out at a 10° angle for 1st order diffraction
- Assume $n_{\text{eff}} = 2.5, \lambda = 1.55\mu m$

$$\sin(\theta) = \frac{n_{\text{eff}} - \frac{\lambda}{a}}{n_{\text{air}}}$$

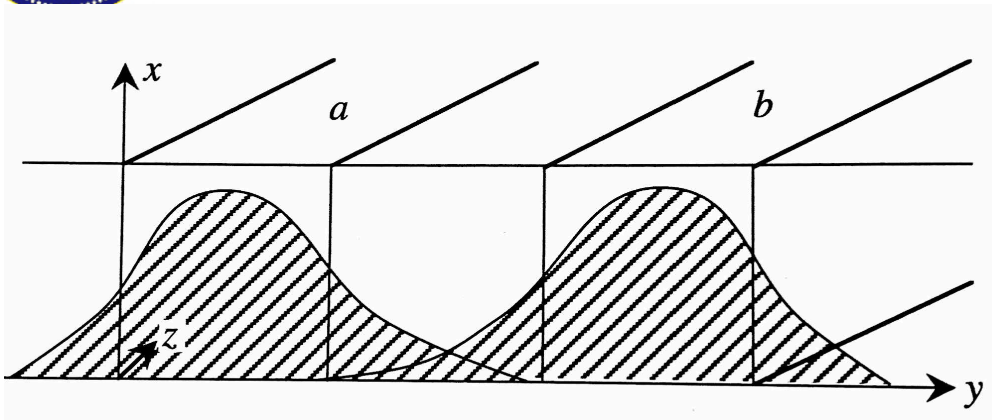
$$\sin(10°) = \frac{2.5 - \frac{1.55}{a}}{1}$$

$$a \sim 670 \text{ nm}$$



What thickness of SiO_2 shall we choose? ← Destructive interference to cancel bottom emission

Directional Coupler



$$\mathbf{E}(x,y,z) = a(z)\mathbf{E}^{(a)}(x,y) + b(z)\mathbf{E}^{(b)}(x,y)$$

Coupled Mode Theory:

$$\begin{cases} \dfrac{da(z)}{dz} = i\beta_a a(z) + iK_{ab} b(z) \\ \dfrac{db(z)}{dz} = iK_{ba} a(z) + i\beta_b b(z) \end{cases}$$

$K_{ab} = K_{ba}^*$ for co-directional coupling

$K_{ab} = -K_{ba}^*$ for counter-directional coupling

- Light can couple from one waveguide to an adjacent waveguide in close proximity
  - Similar to electron tunneling through a barrier
- Optical modes should overlap
- Wave vector needs to match
- The coupling can be described by "coupled mode theory"

Solution for Coupled Mode Equations

$$\frac{d}{dz}\begin{bmatrix} a \\ b \end{bmatrix} = i\mathbf{M}\begin{bmatrix} a \\ b \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} \beta_a & K_{ab} \\ K_{ba} & \beta_b \end{bmatrix}$$

Solution has the form: $\begin{bmatrix} a(z) \\ b(z) \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} e^{i\beta z}$

$$\begin{bmatrix} \beta_a - \beta & K_{ab} \\ K_{ba} & \beta_b - \beta \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = 0$$

Determinant = 0:

$$(\beta_a - \beta)(\beta_b - \beta) - K_{ab}K_{ba} = 0$$

Solution:

$$\beta = \frac{\beta_a + \beta_b}{2} \pm q$$

$$q = \sqrt{\Delta^2 + K_{ab}K_{ba}}$$

$$\Delta = \frac{\beta_a - \beta_b}{2}$$

Special case: identical waveguides

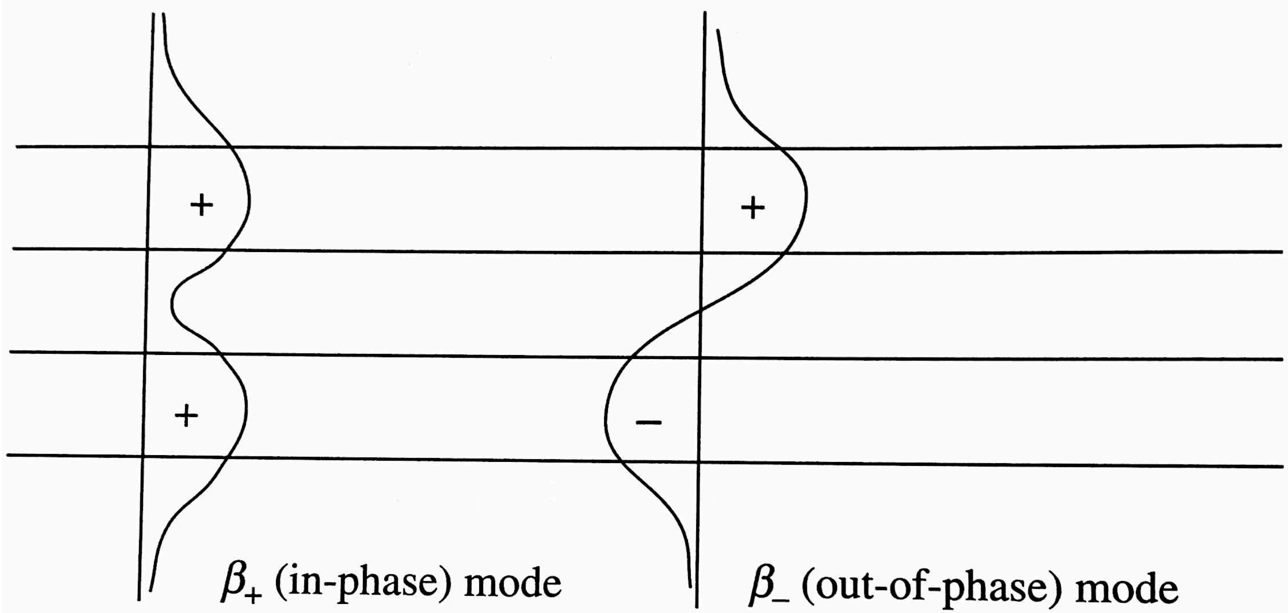
$$\beta_a = \beta_b = \beta_0$$

$$\beta = \beta_0 \pm \sqrt{K_{ab} K_{ba}} = \beta_\pm$$

Eigen modes of the coupled waveguides:

In-phase mode: β_+

Out-of-phase mode: β_-



β_+ (in-phase) mode

β_- (out-of-phase) mode

Special case: identical waveguides

Eigen vectors:

$$\text{For } \beta_+ \ : \ \mathbf{v}_1 = \begin{bmatrix} K_{ab} \\ q \end{bmatrix}$$

$$\text{For } \beta_- \ : \ \mathbf{v}_2 = \begin{bmatrix} K_{ab} \\ -q \end{bmatrix}$$

$$\text{Eigenmatrix:} \quad \mathbf{V} = \left[\mathbf{v}_1 \vdots \mathbf{v}_2 \right] = \begin{bmatrix} K_{ab} & K_{ab} \\ q & -q \end{bmatrix}$$

$$\begin{bmatrix} a(z) \\ b(z) \end{bmatrix} = \mathbf{V} \begin{bmatrix} e^{i\beta_+ z} & 0 \\ 0 & e^{i\beta_- z} \end{bmatrix} \mathbf{V}^{-1} \begin{bmatrix} a(0) \\ b(0) \end{bmatrix}$$

Special case: identical waveguides (cont)

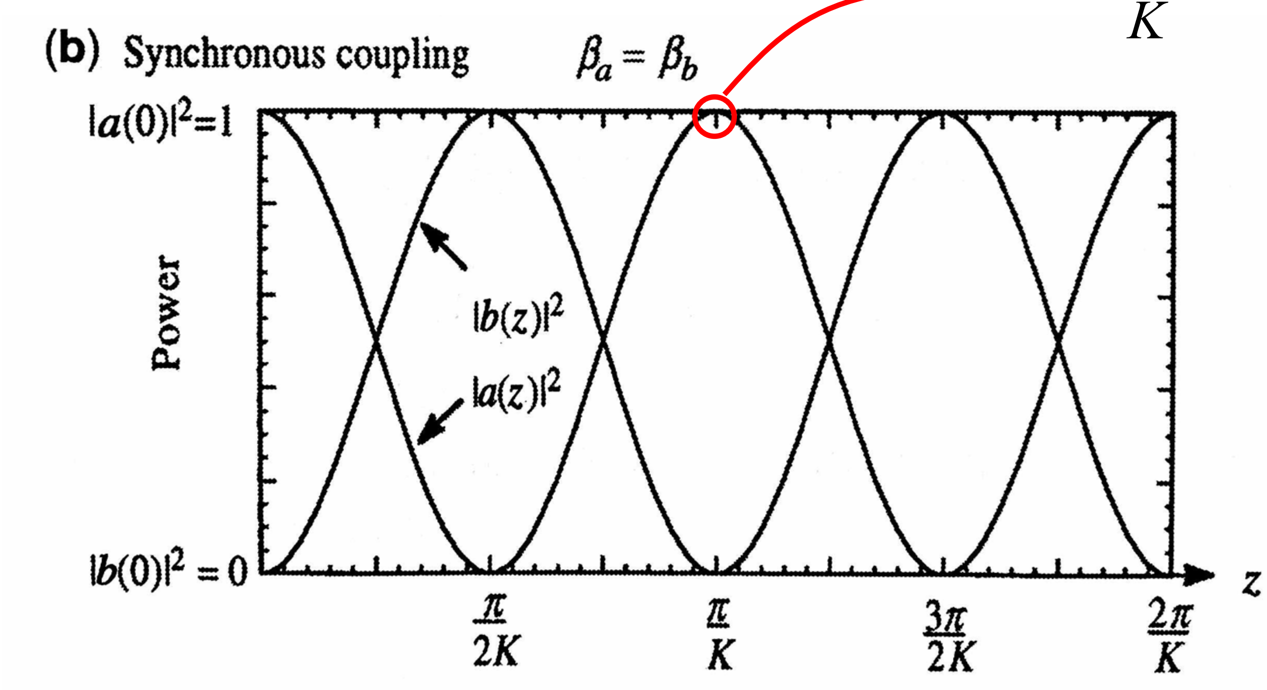
Input from top waveguide: $a(0)=1$ and $b(0)=0$

$$a(z)=\cos(Kz)\cdot e^{i\beta z} \Rightarrow |a(z)|^2=\cos^2(Kz)$$

$$b(z)=i\sin(Kz)\cdot e^{i\beta z} \Rightarrow |b(z)|^2=\sin^2(Kz)$$

Here $\beta=\beta_a=\beta_b$ and $K=K_{ab}=K_{ba}$

$\frac{\pi}{K}$: Beat Length



General Solution: Non-Identical Waveguides

$$\beta_a = \beta_a(\omega) \; ; \qquad \beta_b = \beta_b(\omega)$$

Eigen vectors:

For $\beta_+ : \; \mathbf{v}_1 = \begin{bmatrix} K_{ab} \\ q - \Delta \end{bmatrix}$ For $\beta_- : \; \mathbf{v}_2 = \begin{bmatrix} K_{ab} \\ -q - \Delta \end{bmatrix}$

$$\begin{bmatrix} a(z) \\ b(z) \end{bmatrix} = \mathbf{V} \begin{bmatrix} e^{i\beta_+ z} & 0 \\ 0 & e^{i\beta_- z} \end{bmatrix} \mathbf{V}^{-1} \begin{bmatrix} a(0) \\ b(0) \end{bmatrix}$$

$$= e^{i\phi z} \begin{bmatrix} \cos qz + i\frac{\Delta}{q}\sin qz & i\frac{K_{ab}}{q}\sin qz \\ i\frac{K_{ba}}{q}\sin qz & \cos qz - i\frac{\Delta}{q}\sin qz \end{bmatrix} \begin{bmatrix} a(0) \\ b(0) \end{bmatrix}$$

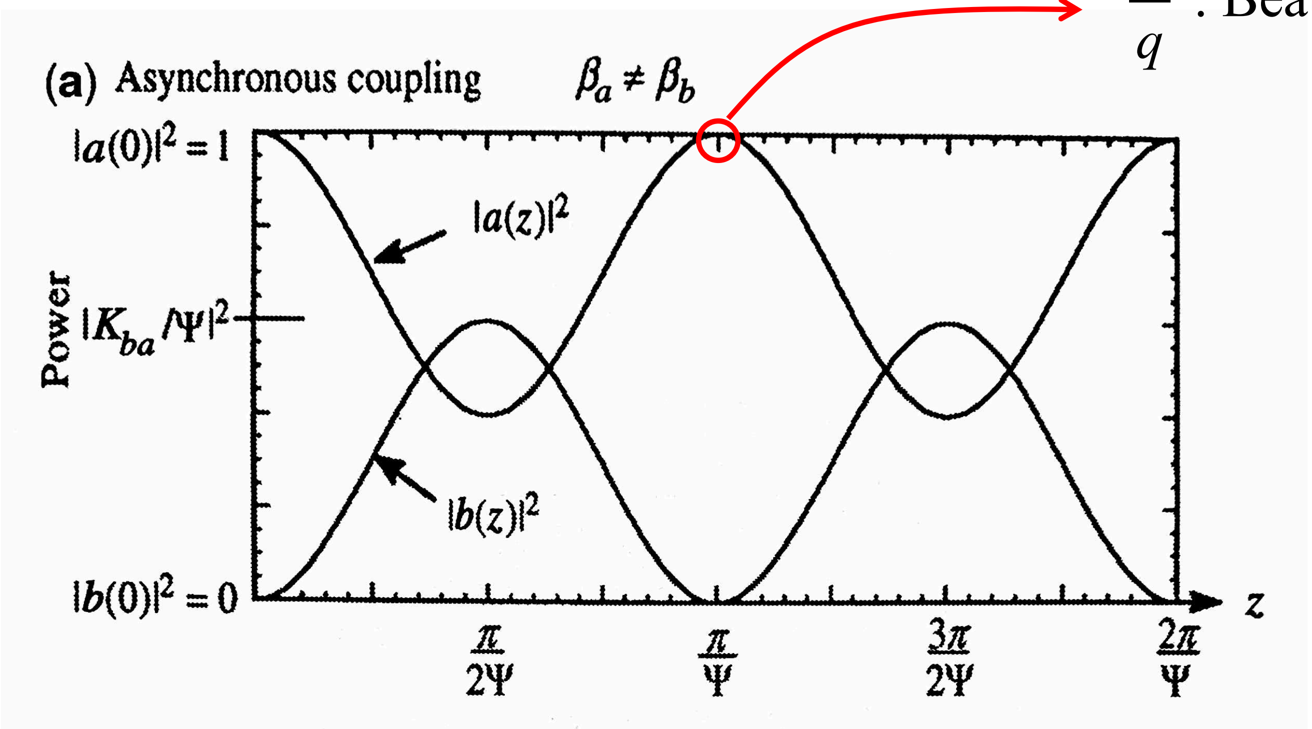
General Solution: Non-Identical Waveguides

Input from top waveguide: $a(0) = 1$ and $b(0) = 0$

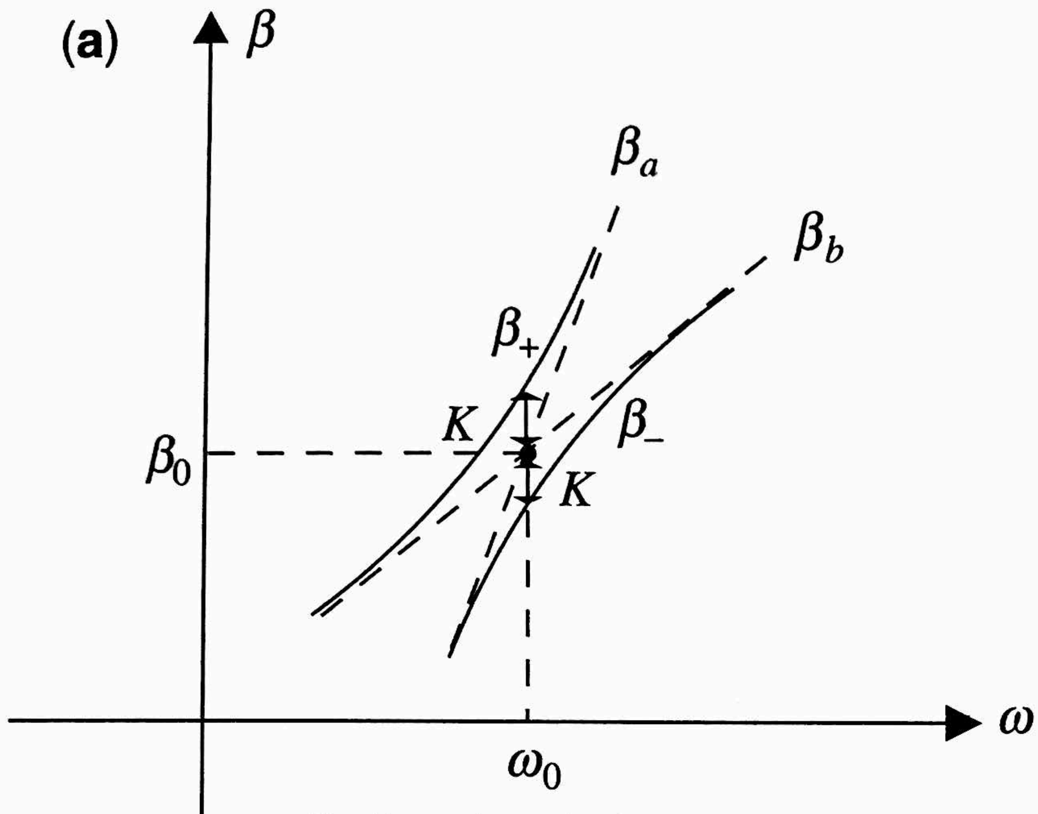
$$\left|b(z)\right|^2 = \left|\frac{K_{ba}}{q}\right|^2 \sin^2(qz)$$

Maximum power transfer: $\left|\frac{K_{ba}}{q}\right|^2 = \frac{\left|K_{ab}\right|^2}{\Delta^2 + \left|K_{ab}\right|^2} < 1$

$\frac{\pi}{q}$: Beat Length



Dispersion Curves for Directional Couplers



Codirectional coupling

Application: Optical Switches

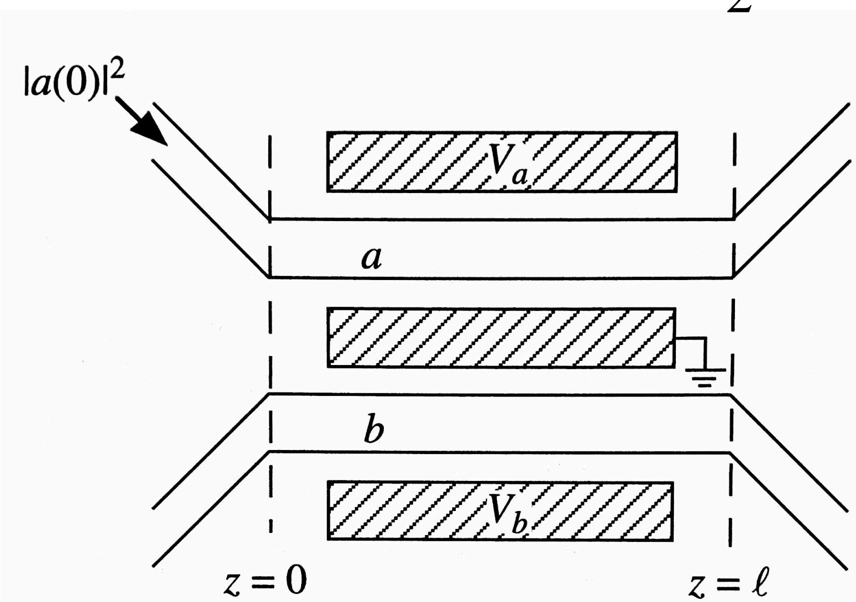
Input from top waveguide: $a(0) = 1$ and $b(0) = 0$

Output from waveguide b :

$$P_b = |b(l)|^2 = \frac{K^2}{\Delta^2 + K^2} \sin^2(ql)$$

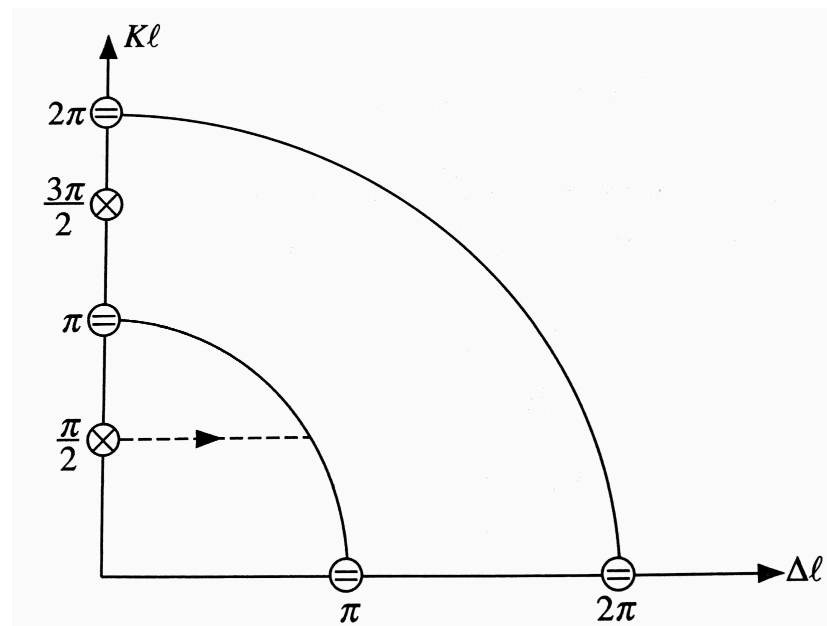
"Bar" state: $ql = n\pi, \quad n = 1,2,3..., \text{ or } \quad (\Delta l)^2 + (Kl)^2 = (n\pi)^2$

"Cross" state: $ql = (n + \frac{1}{2})\pi, \quad n = 1,2,3...$

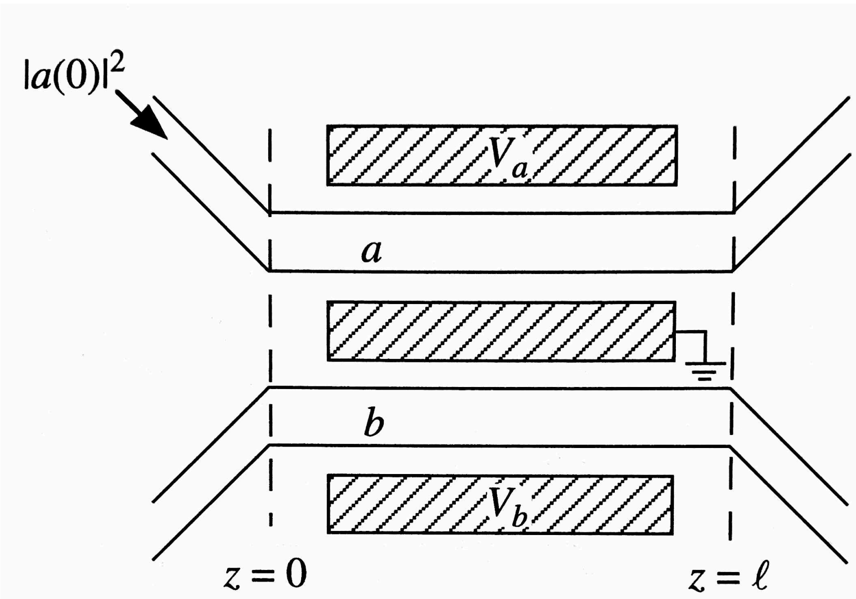


Bar State

Cross State

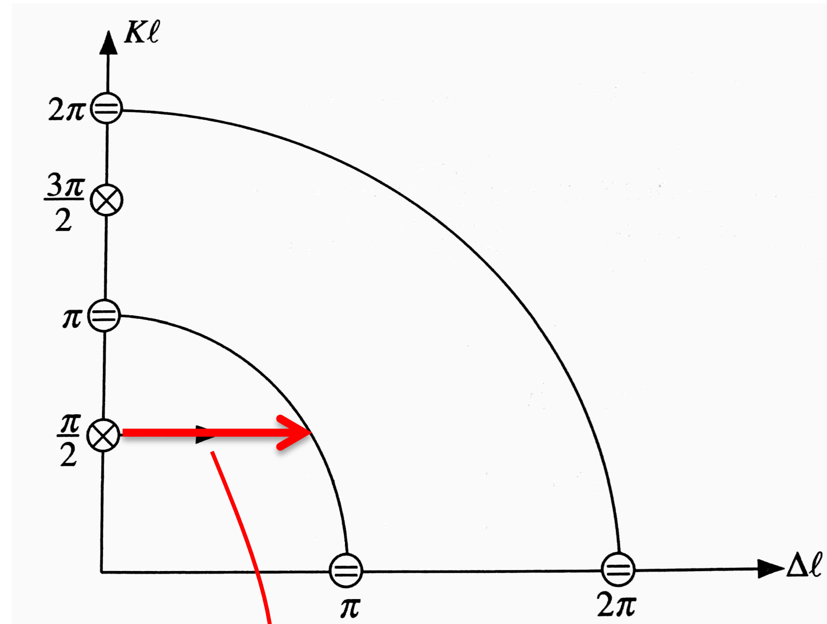


Application: Optical Switches



⊜ Bar State

⊗ Cross State



Switch from Cross to Bar state:

$$\text{Change } \Delta = 0 \text{ to } \Delta = \frac{\pi}{l} = \frac{\beta_a - \beta_b}{2} = \frac{2\pi}{\lambda}(n_a - n_b)$$

$$\text{i.e., change } \Delta n = n_a - n_b = \frac{\lambda}{2l} = \frac{\lambda}{2\cdot(\pi/2K)} = \frac{K\lambda}{\pi}$$

by electro-optic or thermo-optic effect

Improved Switch: