EE 232 Lightwave Devices
Lecture 9: Integrated Photonic Components (3)
Microring Resonator

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Reading: Chuang, Chap 8
Microring Resonators

- Compact resonators for photonic integrated circuits (PIC)
- Small footprint, particularly in Si photonics (high index contrast waveguide)
- Wide range of applications, including modulators, lasers, detectors, filters, switches, dispersion management, nonlinear optical devices (optical frequency combs), ...

Q. Xu, ..., M. Lipson, Nature 2005
C. Sun, .... V.M. Stonanovic, Nature 2015
Coupled Mode Equations for Microring Resonators

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} = \begin{bmatrix}
  t & i\kappa \\
  i\kappa^* & t^*
\end{bmatrix} \begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix}
\]

\[t \cdot t^* + \kappa \cdot \kappa^* = 1\]

\[a_2 = e^{-\alpha L/2 + i\beta_{eff} L} b_2 = a e^{i\theta} b_2 = X \cdot b_2\]

\[
\begin{cases}
  a = e^{-\alpha L/2} : \text{remaining field after one round trip} \\
  \theta = \beta_{eff} L : \text{round-trip phase shift}
\end{cases}
\]

\[b_2 = i\kappa^* a_1 + t^* a_2 = \frac{a_2}{X} \Rightarrow a_2 = \frac{i\kappa^*}{1 - t^*} a_1\]

\[b_1 = t a_1 + i\kappa a_2 = t a_1 + i\kappa \frac{i\kappa^*}{1 - t^*} a_1 = \frac{t}{X} \left( \frac{1}{X} - t^* \right) - |\kappa|^2 a_1 = \frac{t}{X} - 1 \frac{1}{1 - t^* X} = \frac{t - X}{1 - t^* X}\]

\[\frac{b_1}{a_1} = \frac{t - ae^{i\theta}}{1 - at^* e^{i\theta}}\]
Transfer Characteristics for Microring Resonators

\[
T = \left| \frac{b_1}{a_1} \right|^2 = \frac{\left| t \right|^2 + a^2 - 2a \left| t \right| \cos(\theta - \theta_t)}{1 + \left| at \right|^2 - 2a \left| t \right| \cos(\theta - \theta_t)}
\]

Transmission has a dip at resonance:

\[
\theta - \theta_t = 2m\pi = \beta_{eff} L - \theta_t = \frac{2\pi f_m}{c} n_{eff} L - \theta_t
\]

Frequency spacing between consecutive resonance:

\[
\Delta f = FSR = f_{m+1} - f_m = \frac{2\pi}{2\pi n_{eff} L} = \frac{c}{n_{eff} L} : \text{Free spectral range}
\]

Transmission at resonance:

\[
T_{res} = \frac{\left| t \right|^2 + a^2 - 2a \left| t \right|}{1 + \left| at \right|^2 - 2a \left| t \right|} = \frac{\left( \left| t \right| - a \right)^2}{\left( 1 - a \left| t \right| \right)^2}
\]

\[T_{res} = 0 \text{ when } a = \left| t \right| : \text{critical coupling condition}\]
Coupling Regimes of Microring Resonator

At resonance,

\[ T = \left| \frac{b_1}{a_1} \right|^2 = \frac{(|t| - a)^2}{(1 - a|t|)^2} \]

1. Critical coupling: \( a = |t| \)
   destructive interference completely cancels out transmitted light, \( b_1 \)
2. Under coupling: \( a < |t| \)
3. Over coupling: \( a > |t| \)
4. With gain, \( a \) could > 1.

Lasing occurs when \( a = \frac{1}{|t|} \)

Another expression for critical coupling:

\[ 1 - a^2 = 1 - e^{-\alpha L} = \text{round-trip loss} \]

\[ 1 - |t|^2 = |\kappa|^2 \]

1. Critical coupling: coupling = round-trip loss
2. Under coupling: coupling < round-trip loss
3. Over coupling: coupling > round-trip loss
Optical Add-Drop Multiplexer
(4 port device)
Transfer Curves

(a) High Loss

(b) Medium Loss

(c) Low Loss

Input: $a_1$, $t_1$, $\kappa_1$, $b_1$

Output: $d_1$, $c_1$

(Through)

Output: $c_2$, $t_2$, $\kappa_2$, $d_2 = 0$

Transmission:

- $T_1$
- $T_2$
- $T_1 + T_2$

Angle (radian):

- $2m\pi$
- $(2m + 1)\pi$
- $(2m + 2)\pi$

$a = 0.8$

$a = 0.9$

$a = 0.99$
Similarity with Fabry-Perot Resonator

Mirror 1 ($r_1, t_1$)  
Mirror 2 ($r_2, t_2$)

\[ E_{inc} \rightarrow \rightarrow \rightarrow E_r \]  
\[ E_r \leftarrow \leftarrow \leftarrow \rightarrow E_t \]  
\[ z \]

\[ L \]

Transmission

\[ T = \frac{|E_t|^2}{|E_i|^2} = \frac{(1-r_1^2)(1-r_2^2)}{(1-r_1r_2)^2 + 4r_1r_2 \sin^2 kL} \]

\[ R = \frac{|E_r|^2}{|E_i|^2} = 1 - T \]

Wavelength (or Frequency)
Similarity with Fabry-Perot Resonator

Transmission = Drop Port

Reflection = Through Port

Input $a_1$ $t_1, \kappa_1$ $b_1$ Output (Through)

$E_{inc}$ $\rightarrow$ $E_r$

$E_t$ $\leftarrow$