# EE 232 Lightwave Devices
# Lecture 11: Absorption in bulk semiconductors


Instructor: Ming C. Wu

University of California, Berkeley
Electrical Engineering and Computer Sciences Dept.



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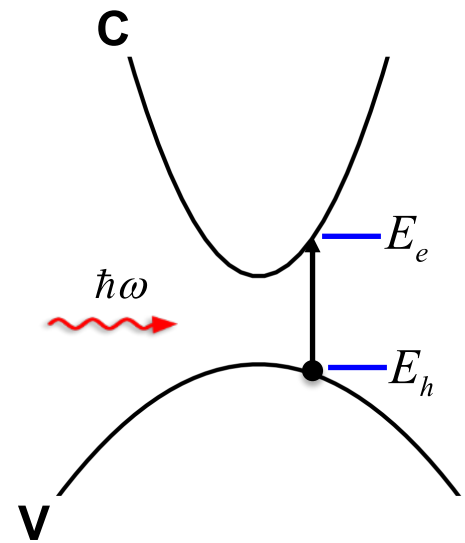
# Absorption / Emission Rates

Consider transition rate for between a state in valence band and a state in conduction band:

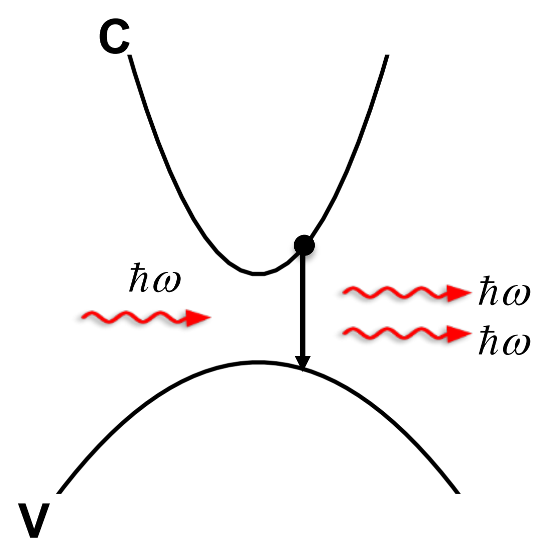



<u>Absorption</u>

$$w_{abs} = \frac{2\pi}{\hbar^2}\left|H'_{fi}\right|^2 \delta(\omega_{fi} - \omega)$$

$$= \frac{2\pi}{\hbar}\left|H'_{cv}\right|^2 \delta(E_e - E_h - \hbar\omega)$$

<u>Stimulated emission</u>




$$w_{ems} = \frac{2\pi}{\hbar^2}\left|H'_{if}\right|^2 \delta(\omega_{if} - \omega)$$

$$= \frac{2\pi}{\hbar}\left|H'_{cv}\right|^2 \delta(E_e - E_h - \hbar\omega)$$

Note: change of variable for delta function

$$\delta(ax) = \frac{1}{a}\delta(x)\ ; \qquad \delta(E) = \delta(\hbar\omega) = \frac{1}{\hbar}\delta(\omega)$$

# Net Absorption Rate from VB to CB

## in the presence of carriers (from doping or pumping)

Now, for a semiconductor we have a continuum of final and initial states. Upward transition rate considering the probability that the valence band state is filled and the conduction band state is empty is given by

$$R_{v\to c} = \frac{2}{V}\sum_{k_c=k_v}\frac{2\pi}{\hbar}\left|\hat{H}_{cv}\right|^2\delta(E_e - E_h - \hbar\omega) f_v (1 - f_c) \quad \left(\mathrm{s}^{-1}\mathrm{cm}^{-3}\right)$$

Similarly, the downward transition rate,

$$R_{c\to v} = \frac{2}{V}\sum_{k_c=k_v}\frac{2\pi}{\hbar}\left|\hat{H}_{cv}\right|^2\delta(E_e - E_h - \hbar\omega) f_c (1 - f_v) \quad \left(\mathrm{s}^{-1}\mathrm{cm}^{-3}\right)$$

The total net upward rate is then,

$$R = R_{v\to c} - R_{c\to v} = \frac{2}{V}\sum_{k_c=k_v}\frac{2\pi}{\hbar}\left|\hat{H}_{cv}\right|^2\delta(E_e - E_h - \hbar\omega)\left(f_v - f_c\right) \quad \left(\mathrm{s}^{-1}\mathrm{cm}^{-3}\right)$$

# Relation between Absorption Coefficient and Absorption Rate

$I(z)$: optical intensity $= \dfrac{\varepsilon E^2}{2}\dfrac{c}{n}$ unit: $[\dfrac{\text{J}}{\text{m}^3}\dfrac{\text{m}}{\text{s}}]=[\dfrac{\text{J}}{\text{m}^2\text{s}}]$

$I(z) = I_0 e^{-\alpha z}$ ; $\alpha$: absorption coefficient in the media

Number of photons absorbed between $z$ and $z+\Delta z$ :

$$I(z) - I(z+\Delta z) = -\frac{dI(z)}{dz}\Delta z = \alpha I(z)\Delta z$$

$$\alpha = \left(\frac{I(z)-I(z+\Delta z)}{\hbar\omega\cdot\Delta z}\right)\frac{1}{\left(\dfrac{I(z)}{\hbar\omega}\right)} = \frac{\text{No. absorbed photons per unit volume}}{\text{No. of incoming photons per sec. per unit area}}$$

$$\alpha = \frac{R}{I/\hbar\omega} = \frac{\hbar\omega}{\dfrac{c}{n}\dfrac{\epsilon_0\epsilon_r E_0^2}{2}}R$$

$$\alpha = \frac{\hbar\omega}{\dfrac{c}{n}\dfrac{\epsilon_0\epsilon_r E_0^2}{2}}\frac{2}{V}\sum_{k_c=k_v}\frac{2\pi}{\hbar}\left|\hat{H}_{cv}\right|^2\delta(E_e - E_h - \hbar\omega)\left(f_v - f_c\right)$$

# Absorption Coefficient

$$\alpha = \frac{\hbar\omega}{\frac{c}{n}\frac{\epsilon_0\epsilon_r E_0^2}{2}} \frac{2}{V}\sum_{k_c=k_v}\frac{2\pi}{\hbar}\left|\hat{H}_{cv}\right|^2 \delta(E_e - E_h - \hbar\omega)\left(f_v - f_c\right)$$

Previously we used dipole approximation:

$$H' = \frac{q\mathbf{r}\cdot\hat{e}E_0}{2} \; ; \qquad \left|\hat{H}_{cv}\right|^2 = \frac{q^2E_0^2\left|\hat{e}\cdot\mathbf{r}_{cv}\right|^2}{4}$$

$$\alpha = \frac{\hbar\omega}{\frac{c}{n}\frac{\epsilon_0 n^2 E_0^2}{2}} \frac{q^2E_0^2}{4}\frac{2}{V}\sum_{k_c=k_v}\frac{2\pi}{\hbar}\left|\hat{e}\cdot\mathbf{r}_{cv}\right|^2 \delta(E_e - E_h - \hbar\omega)\left(f_v - f_c\right)$$

$$\alpha = \left(\frac{\hbar\omega q^2}{2nc\epsilon_0}\frac{2\pi}{\hbar}\right)\left|\hat{e}\cdot\mathbf{r}_{cv}\right|^2\left(\frac{2}{V}\sum_{k_c=k_v}\delta(E_e - E_h - \hbar\omega)\right)\left(f_v - f_c\right)$$

$$\alpha = \left(\frac{\pi\omega q^2}{nc\epsilon_0}\right)\left|\hat{e}\cdot\mathbf{r}_{cv}\right|^2\left(\frac{2}{V}\sum_{k_c=k_v}\delta(E_e - E_h - \hbar\omega)\right)\left(f_v - f_c\right)$$

$$\alpha = C_1\left|\hat{e}\cdot\mathbf{r}_{cv}\right|^2\left(\frac{2}{V}\sum_{k_c=k_v}\delta(E_e - E_h - \hbar\omega)\right)\left(f_v - f_c\right)$$

# Reduced density of states

Reduced Density of States:

(also called Joint Optical Density of States)

$$\frac{2}{V}\sum_{k_c=k_v}\delta(E_e-E_h-\hbar\omega)\rightarrow\int\rho_r(E)dE$$

$$\left.\begin{aligned}E_e&=E_C+\frac{\hbar^2k^2}{2m_e^*}\\E_h&=E_V-\frac{\hbar^2k^2}{2m_h^*}\end{aligned}\right\}\rightarrow\quad E_e-E_h=E_g+\frac{\hbar^2k^2}{2m_r^*}=\hbar\omega$$

$$\text{where}\quad\frac{1}{m_r^*}=\frac{1}{m_e^*}+\frac{1}{m_h^*}$$

$$\frac{\hbar^2k^2}{2m_r^*}=\hbar\omega-E_g$$

← Note the similarity →

$$\rho_r(\hbar\omega)=\frac{1}{2\pi^2}\left(\frac{2m_r^*}{\hbar^2}\right)^{3/2}\sqrt{\hbar\omega-E_g}$$

Previously, we derived

Electron Density of States:

$$\frac{2}{V}\sum_{k_c}\delta(E-E_e)\rightarrow\int\rho_e(E)dE$$

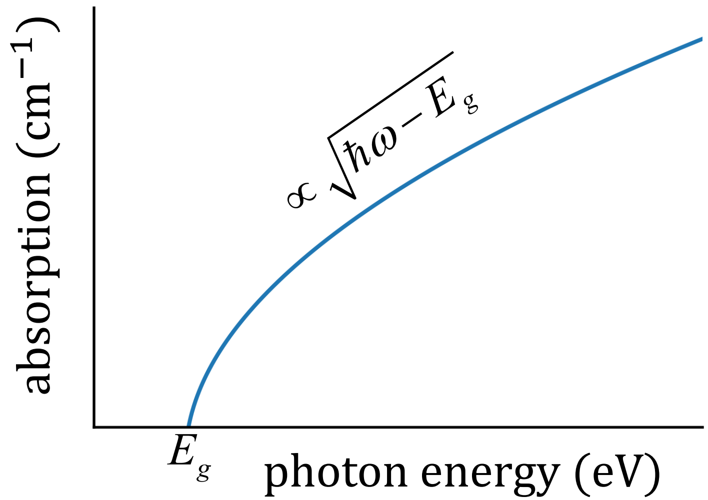
$$E_e=E_C+\frac{\hbar^2k^2}{2m_e^*}$$

$$\frac{\hbar^2k^2}{2m_e^*}=E_e-E_C$$

$$\rho_e(E_e)=\frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}\sqrt{E_e-E_C}$$

# Absorption coefficient

$$\alpha(\hbar\omega) = C_1 \left| \hat{e} \cdot \mathbf{r}_{cv} \right|^2 \rho_r(\hbar\omega) \cdot (f_v - f_c)$$




The absorption coefficient is proportional to

(1) square of matrix element (dipole moment);

(2) reduced density of states (related to effective masses);

(3) Fermi factor, $(f_v - f_c)$



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# Fermi factor

$$f_c(E_e) = \frac{1}{1+\exp[(E_e - F_c)/kT]}$$

$$f_v(E_h) = \frac{1}{1+\exp[(E_h - F_v)/kT]}$$

**We need a change of variables from** $E_e, E_h \to E = \dfrac{\hbar^2 k^2}{2m_r^*}$

$$E = \hbar\omega - E_g = \frac{\hbar^2 k^2}{2m_r^*} \to k = \sqrt{\frac{2m_r^*}{\hbar^2}(\hbar\omega - E_g)}$$

$$E_e = E_g + \frac{\hbar^2 k^2}{2m_e^*}$$

$$= E_g + (\hbar\omega - E_g)\left(\frac{m_r^*}{m_e^*}\right)$$

$$f_c(\hbar\omega) = \frac{1}{1+\exp[(E_g + (\hbar\omega - E_g)m_r^*/m_e^* - F_c)/kT]}$$

$$E_h = -\frac{\hbar^2 k^2}{2m_h^*}$$

$$= -(\hbar\omega - E_g)\left(\frac{m_r^*}{m_h^*}\right)$$

$$f_v(\hbar\omega) = \frac{1}{1+\exp[(-(\hbar\omega - E_g)m_r^*/m_h^* - F_v)/kT]}$$

# Summary

$$\boxed{\alpha(\hbar\omega) = C_1 \left|\hat{e}\cdot\mathbf{r}_{cv}\right|^2 \rho_r(\hbar\omega)\cdot(f_v - f_c)}$$

$$C_1 = \frac{\pi\omega q^2}{nc\epsilon_0} \quad ; \qquad \mathbf{r}_{cv} = \langle \psi_c | \mathbf{r} | \psi_v \rangle$$

$$\rho_r(\hbar\omega) = \frac{1}{2\pi^2}\left(\frac{2m_r^*}{\hbar^2}\right)^{3/2}\sqrt{\hbar\omega - E_g}$$

$$\begin{cases} f_c(\hbar\omega) = \dfrac{1}{1+\exp[(E_g + (\hbar\omega - E_g)m_r^*/m_e^* - F_c)/kT]} \\ f_v(\hbar\omega) = \dfrac{1}{1+\exp[(-(\hbar\omega - E_g)m_r^*/m_h^* - F_v)/kT]} \end{cases}$$

# Comparison with measured data

M.D. Sturge. Phys. Rev. 127, 768 (1963)

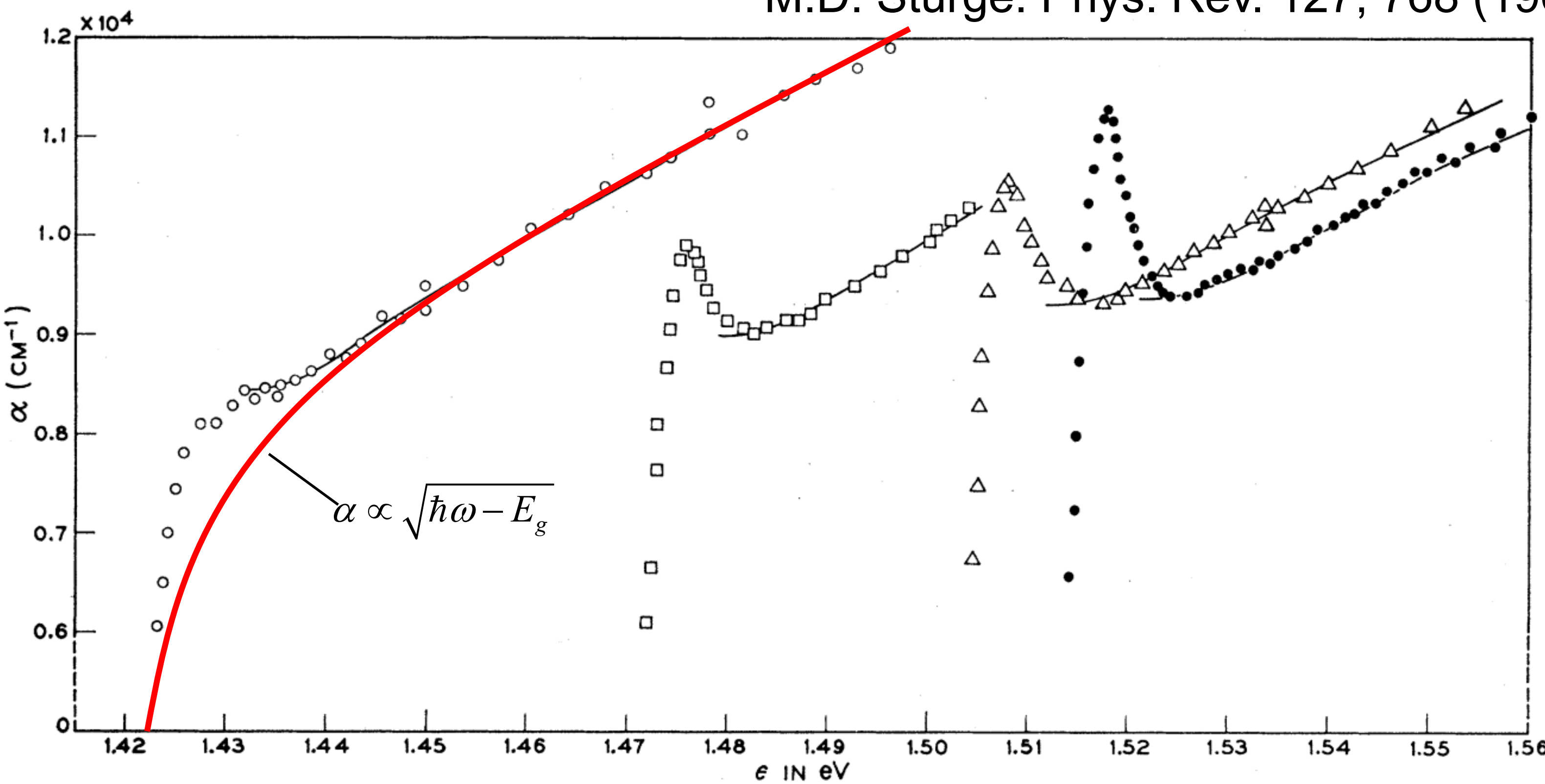


FIG 3 Exciton absorption in GaAs; ○ 294°K, □ 186°K, Δ90°K, ● 21°K.

Absorption follows square root dependence fairly well at room temperature. Our simple model did not include Coulombic interaction between electron and hole which leads to excitonic effect and enhancement of absorption particularly near the bandedge.