EE 232 Lightwave Devices
Lecture 13: Gain in bulk semiconductors

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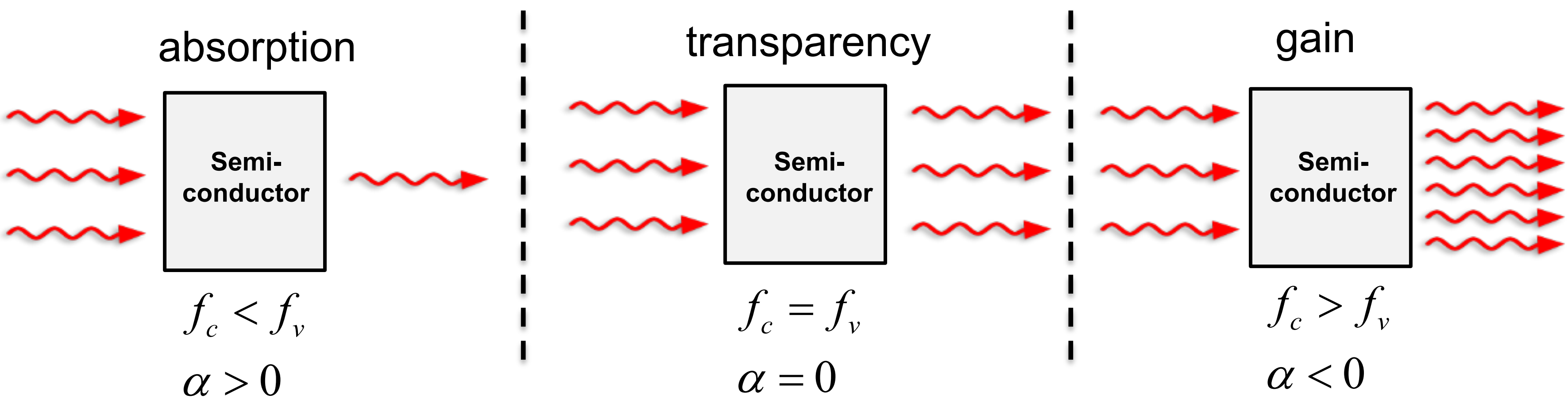
Gain in a semiconductor

$$\alpha(\hbar\omega) = C_0 M_b^2 \rho_r(\hbar\omega)\left[f_v(\hbar\omega) - f_c(\hbar\omega)\right]$$

This expression implies that absorption becomes negative if $f_c > f_v$
Negative absorption is simply gain. We get more photons out than what we put in.

$f_c = f_v$ is known as the transparency condition. Gain is precisely balanced by absorption and the materials is "transparent".



Transparency condition

$$f_c = \frac{1}{1+\exp[(E_g + (\hbar\omega - E_g) m_r^* / m_e^* - F_c)/kT]}$$

$$f_v = \frac{1}{1+\exp[(-(\hbar\omega - E_g) m_r^* / m_h^* - F_v)/kT]}$$

$$f_c = f_v \rightarrow E_g + (\hbar\omega - E_g) m_r^* / m_e^* - F_c = -(\hbar\omega - E_g) m_r^* / m_h^* - F_v$$

$$F_c - F_v = E_g + (\hbar\omega - E_g) m_r^* / m_e^* + (\hbar\omega - E_g) m_r^* / m_h^*$$

$$= E_g + (\hbar\omega - E_g)\left(m_r^* \frac{m_e^* + m_h^*}{m_e^* m_h^*} \right)$$

$$= E_g + (\hbar\omega - E_g)(1)$$

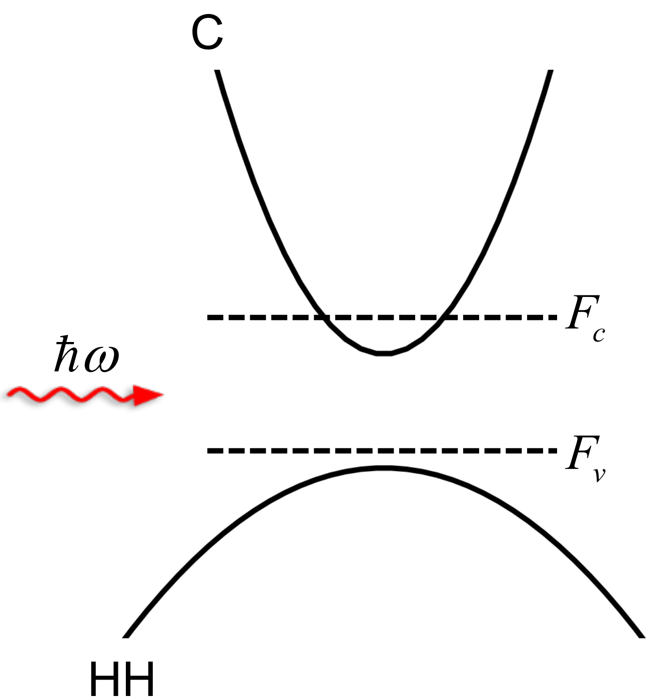
$$F_c - F_v = \hbar\omega$$

$F_c - F_v < \hbar\omega$ (absorption)

$F_c - F_v = \hbar\omega$ (transparency)

$F_c - F_v > \hbar\omega$ (gain)

Transparency carrier concentration (estimation)



Conduction band:
Quasi-Fermi level can easily push into the band such that Boltzmann approx. is not valid.

$$n = N_c F_{1/2}\left(\frac{F_c - E_c}{kT}\right)$$

$$\sim N_c \frac{4}{3\sqrt{\pi}}\left(\frac{F_c - E_c}{kT}\right)^{3/2} \quad \text{for } F_C - E_c \gg kT$$

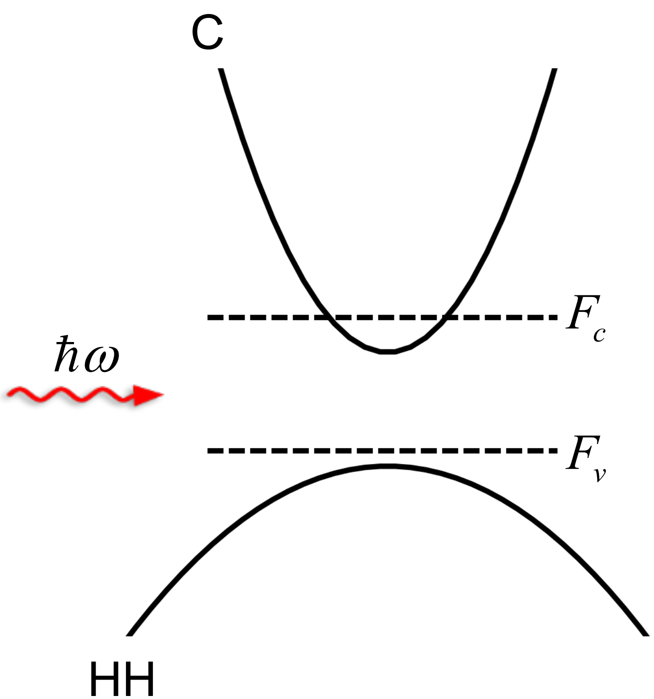
$$F_c \sim n_{tr}^{2/3}\left(\frac{3\sqrt{\pi}}{4N_c}\right)^{2/3} + E_c$$

Valence band:
At transparency, Quasi-Fermi level does not easily push into band. Boltzmann approx. is often adequate

$$p = N_v \exp\left(\frac{E_v - F_v}{kT}\right)$$

$$F_v = E_v - kT\ln\left(\frac{p_{tr}}{N_v}\right)$$

Transparency carrier concentration (estimation)



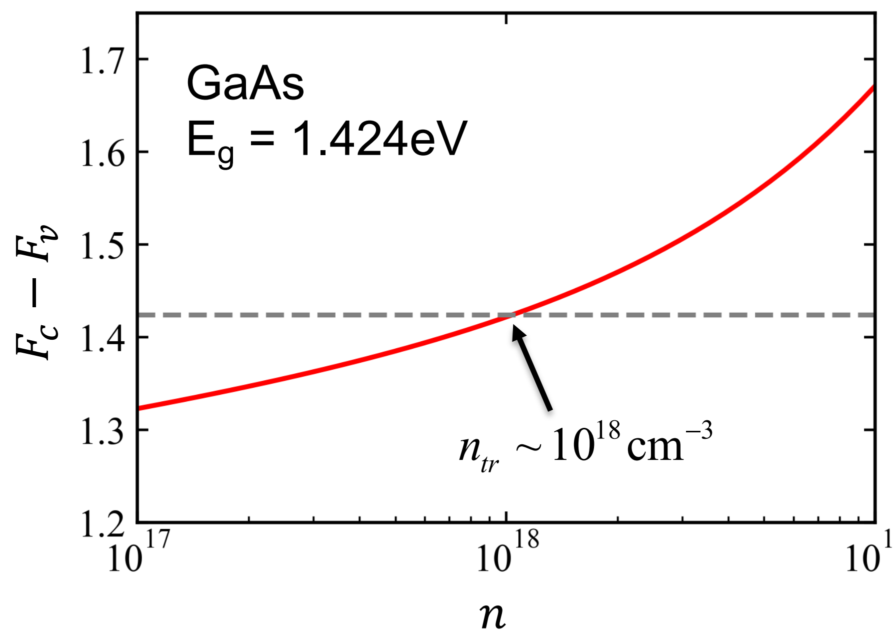
Assume quasi-neutrality applies

$$n = p \text{ and } F_c - F_v = \hbar\omega = E_g$$

$$\rightarrow \quad kTn_{tr}^{2/3}\left(\frac{3\sqrt{\pi}}{4N_c}\right)^{2/3} + E_c - E_v + kT\ln\left(\frac{n_{tr}}{N_v}\right) = E_g$$

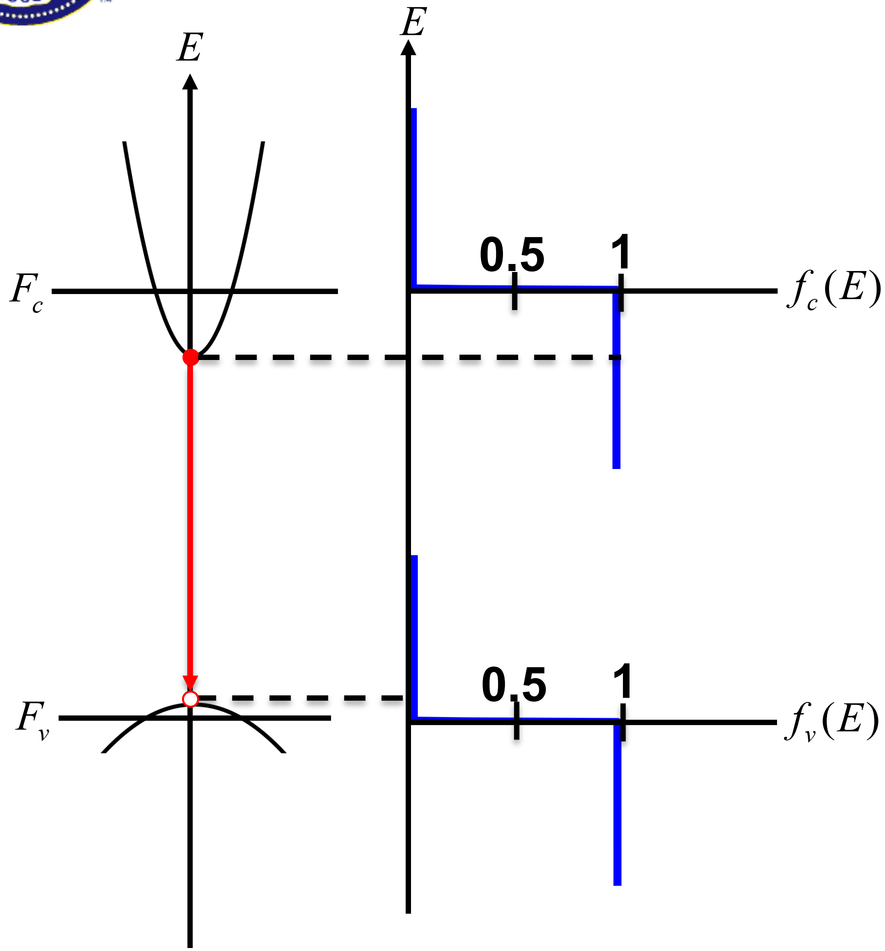
$$n_{tr}^{2/3}\left(\frac{3\sqrt{\pi}}{4N_c}\right)^{2/3} = \ln\left(\frac{N_v}{n_{tr}}\right)$$

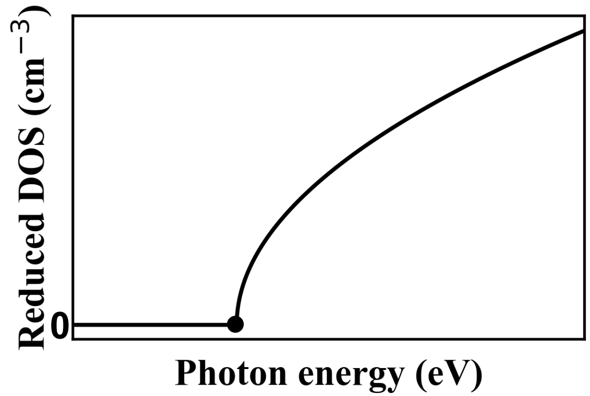
Solve graphically

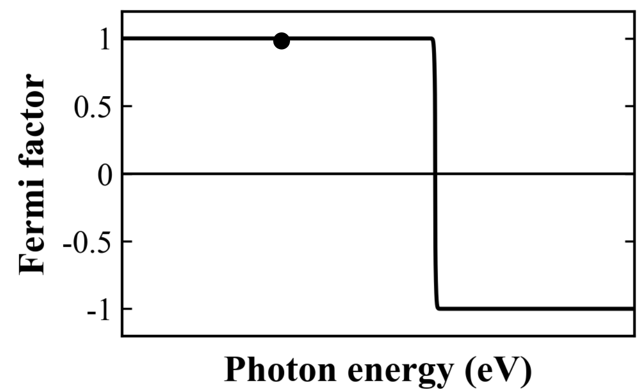


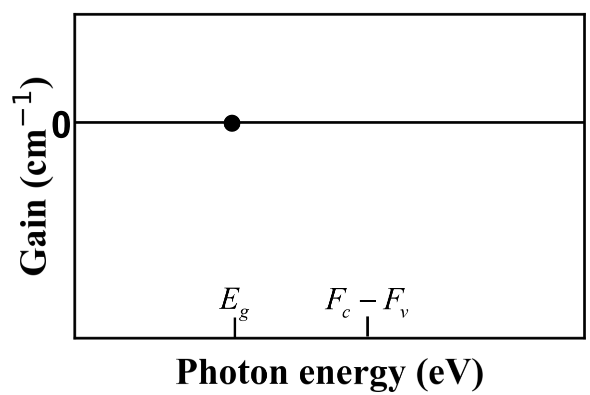
Note that n_{tr} depends only weakly upon effective mass therefore transparency carrier concentration is similar for different direct-bandgap semiconductors.

Gain spectrum (T=0K)





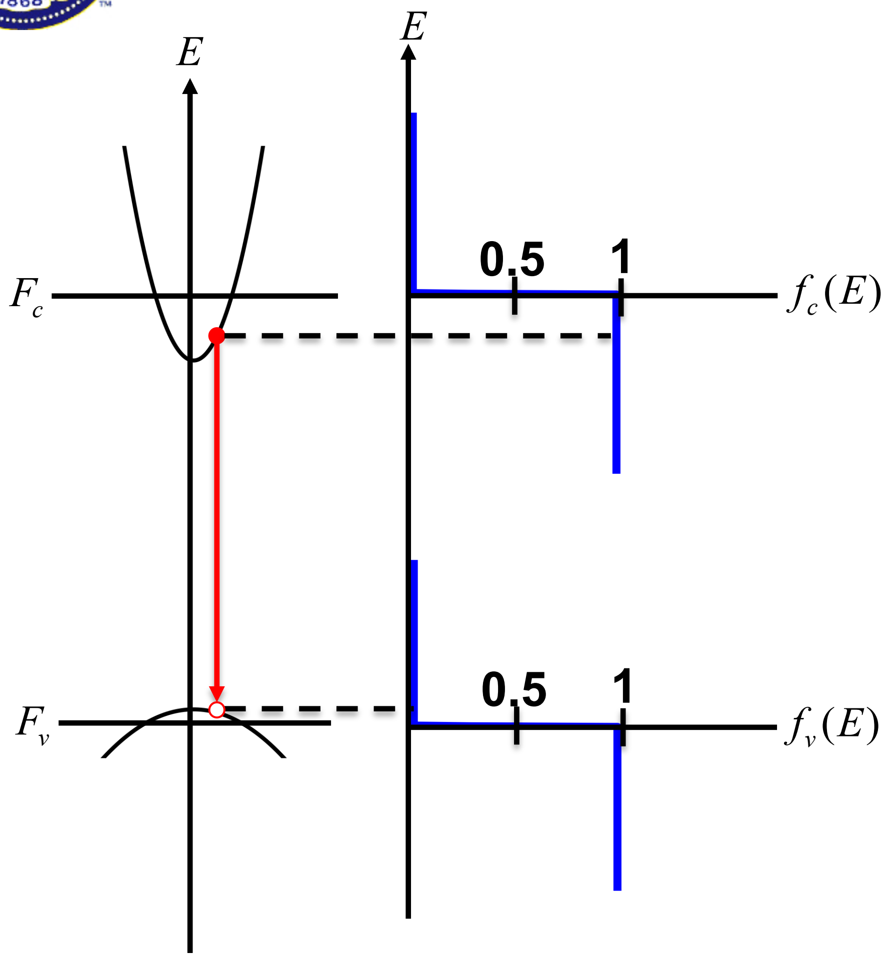


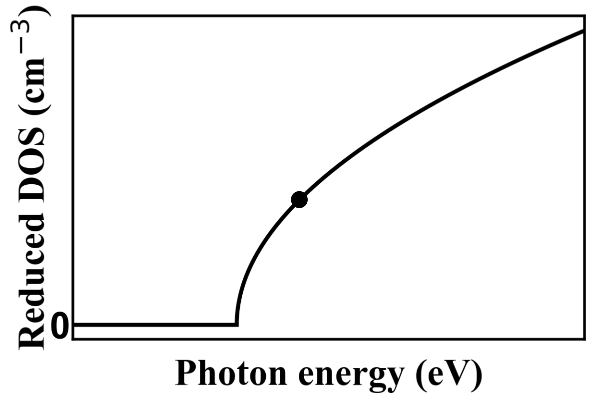


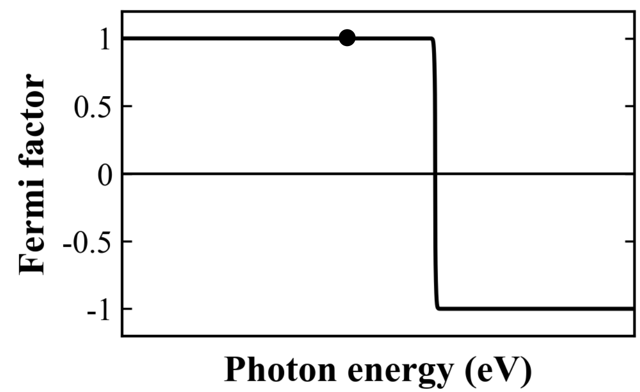
$$g(\hbar\omega) = -\alpha(\hbar\omega)$$
$$= C_0 M_b^2 \rho_r(\hbar\omega)\left[f_c(\hbar\omega) - f_v(\hbar\omega)\right]$$

Fermi inversion factor

Gain spectrum (T=0K)







$$g(\hbar\omega) = -\alpha(\hbar\omega)$$

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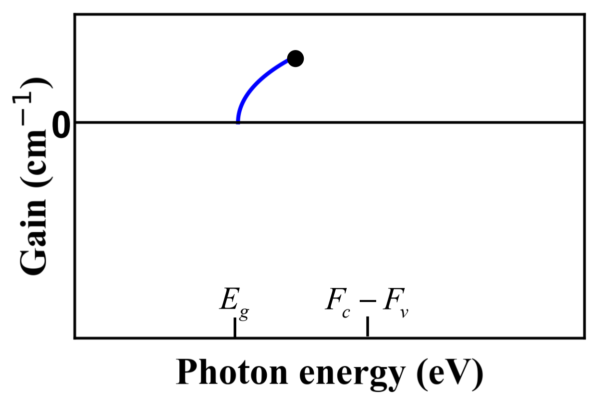
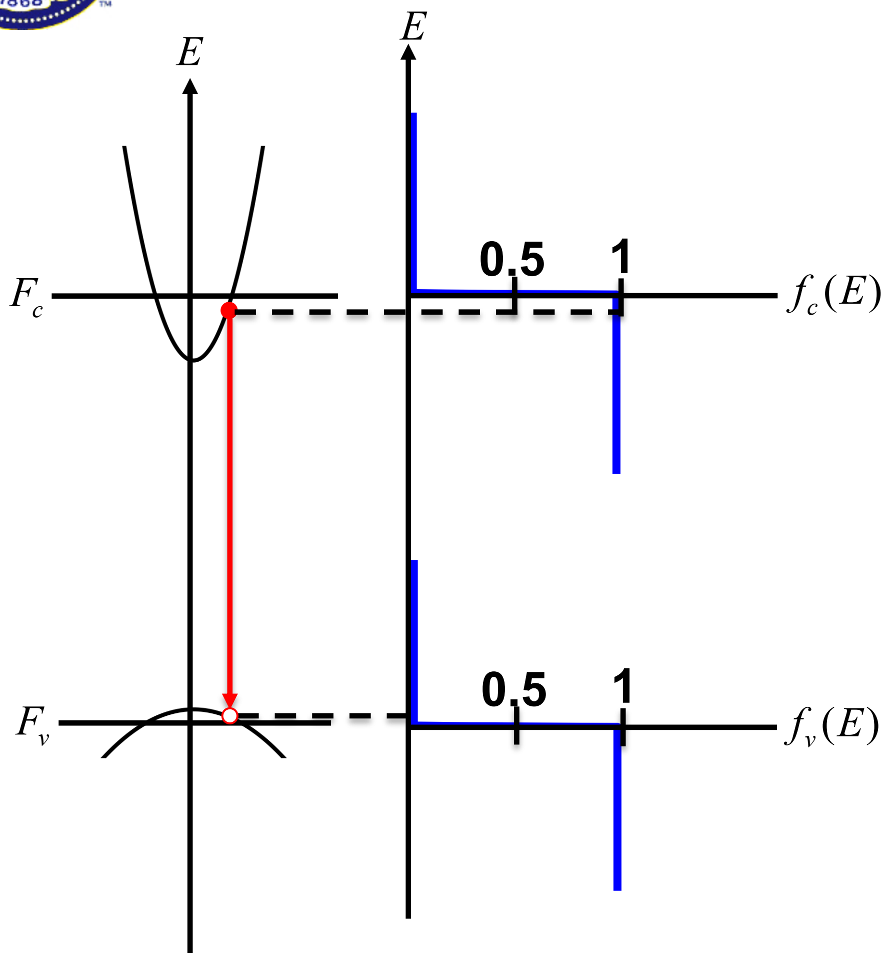
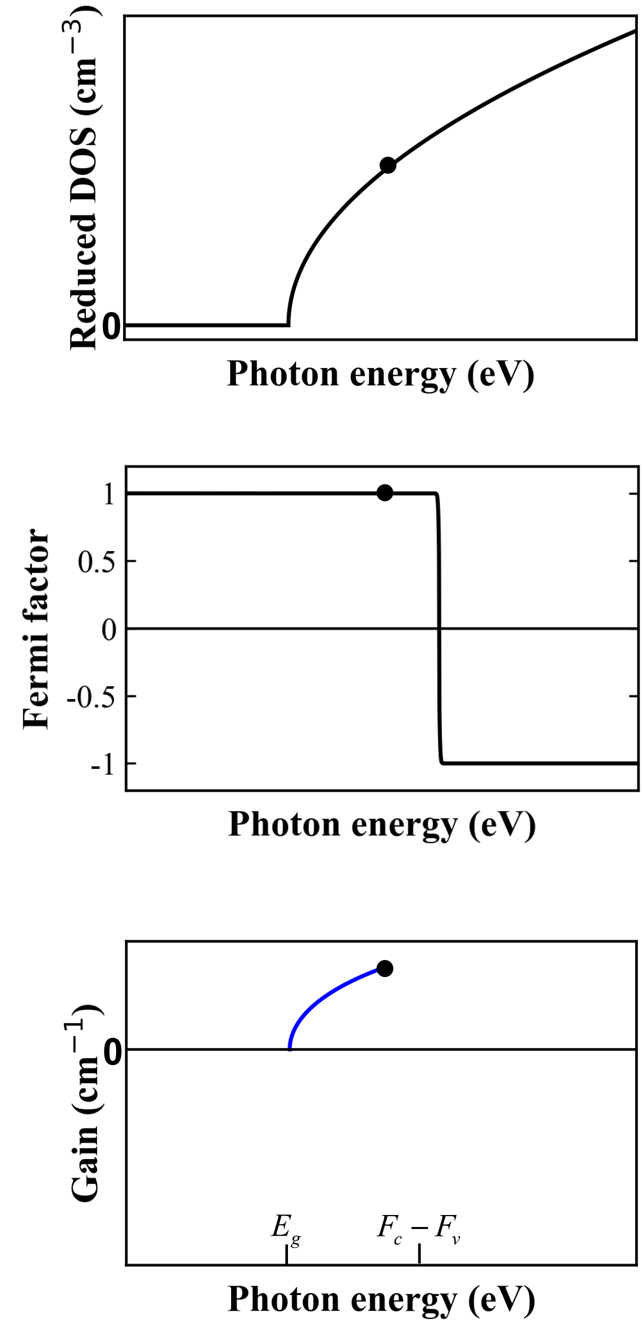
Fermi inversion factor



Gain spectrum (T=0K)





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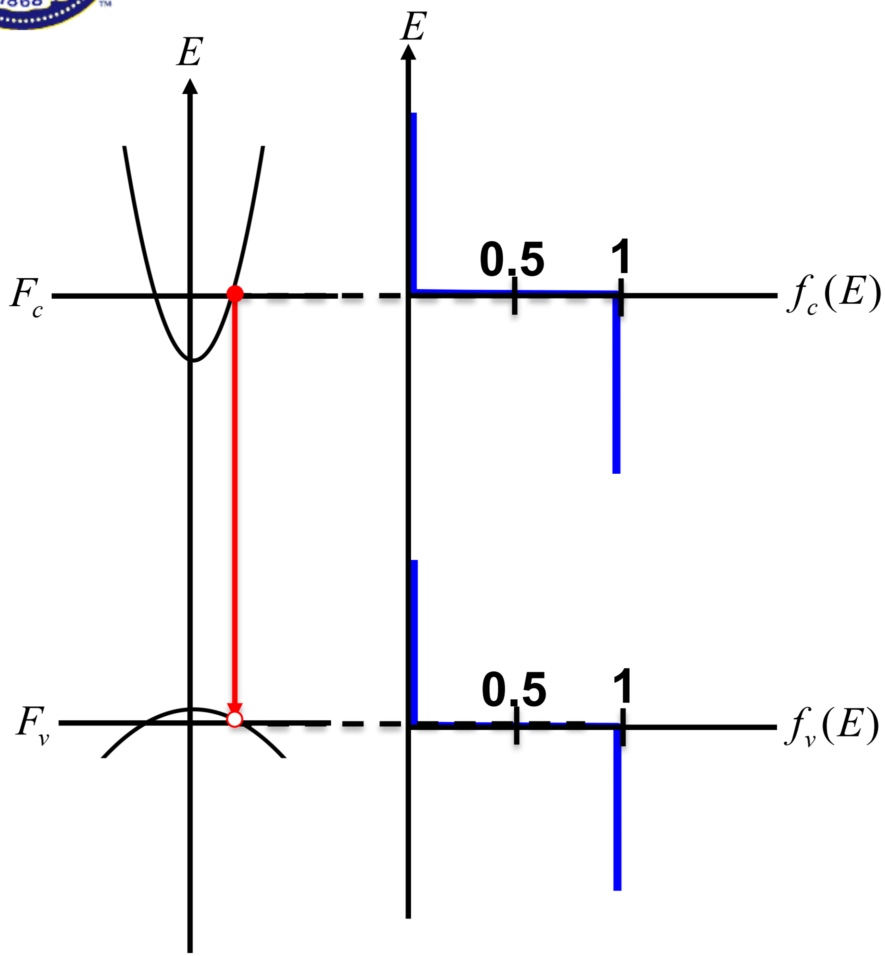

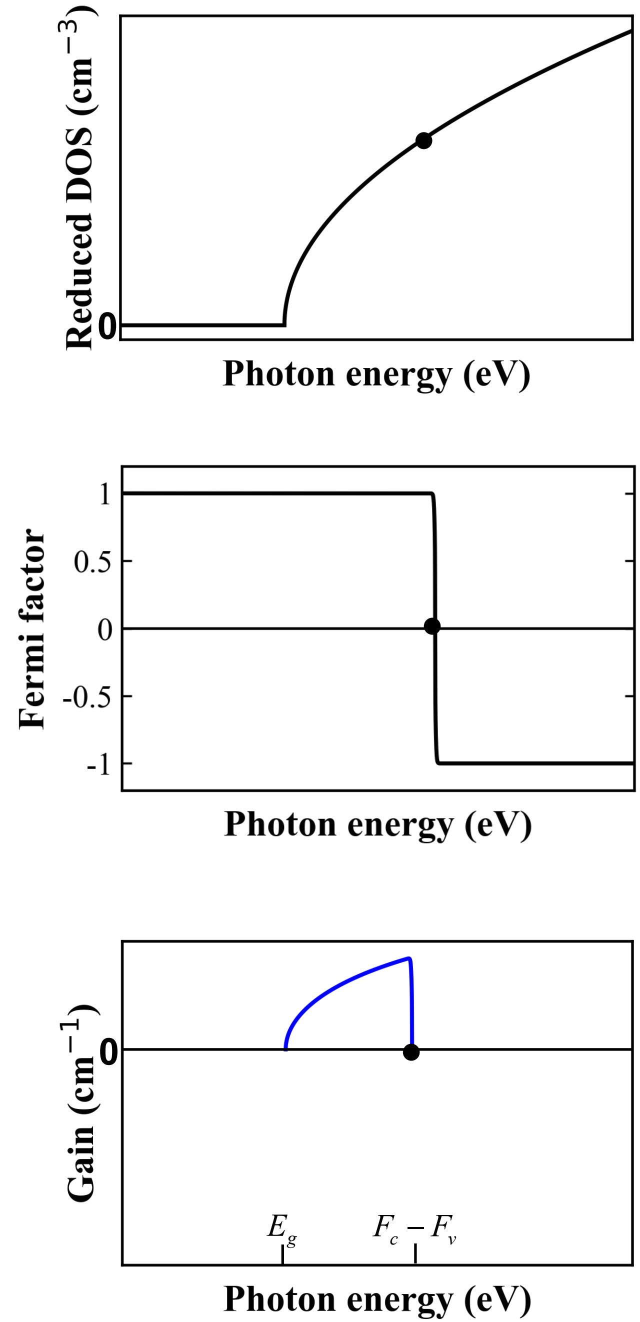

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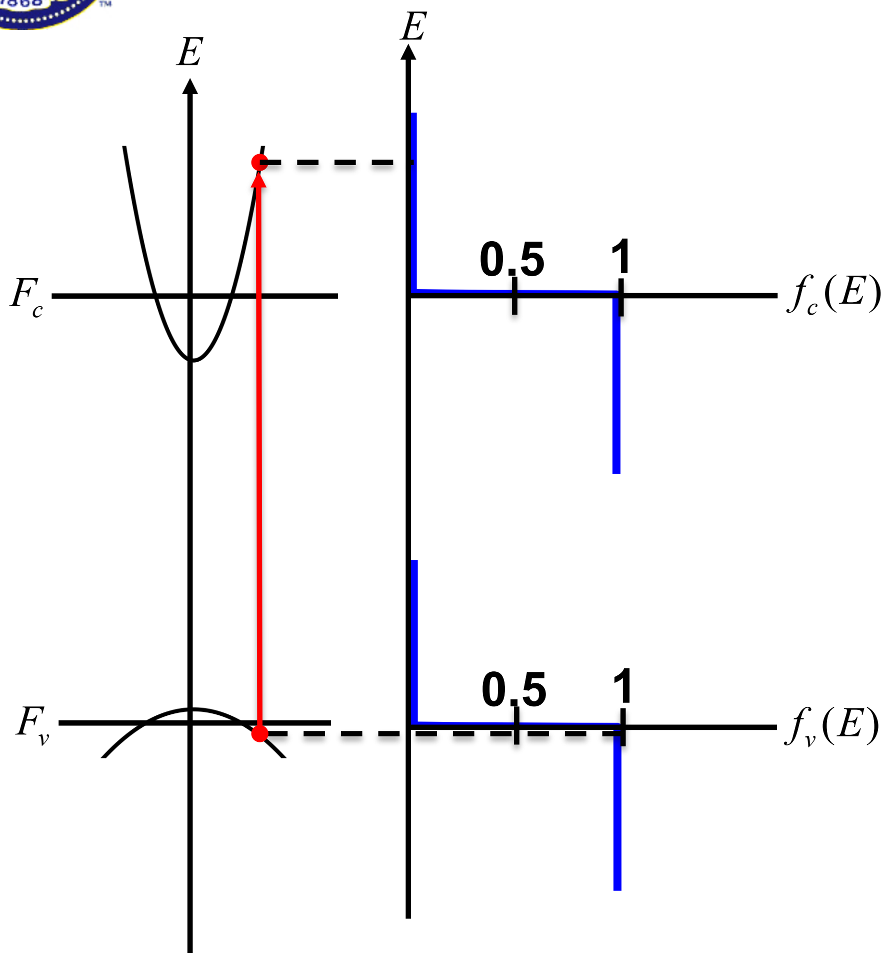

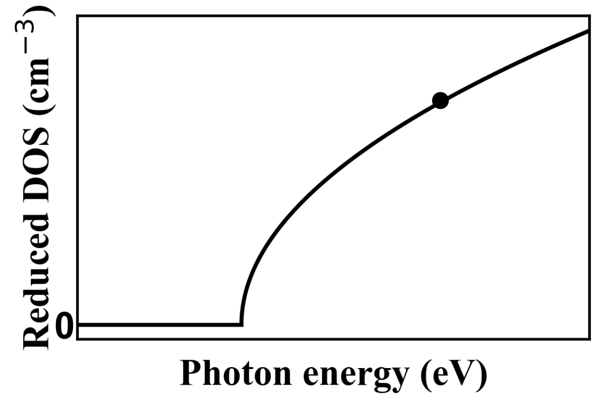

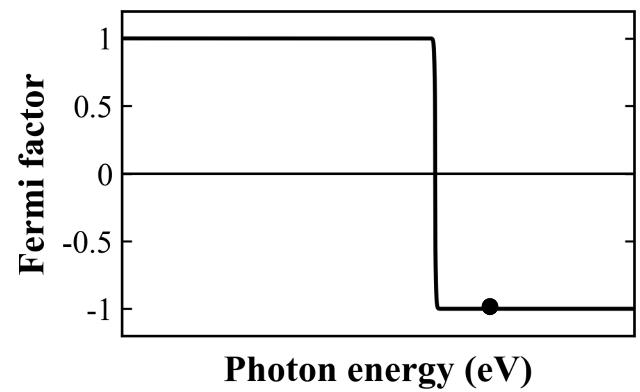

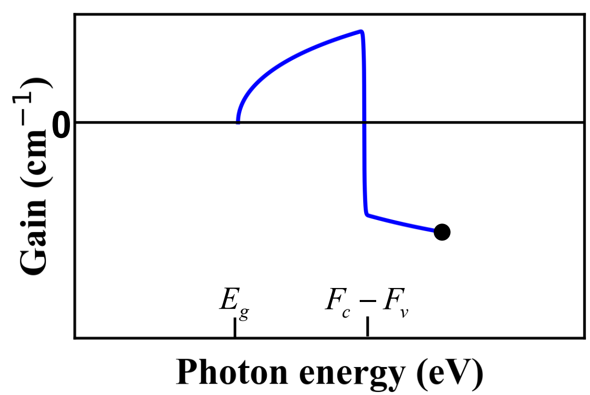

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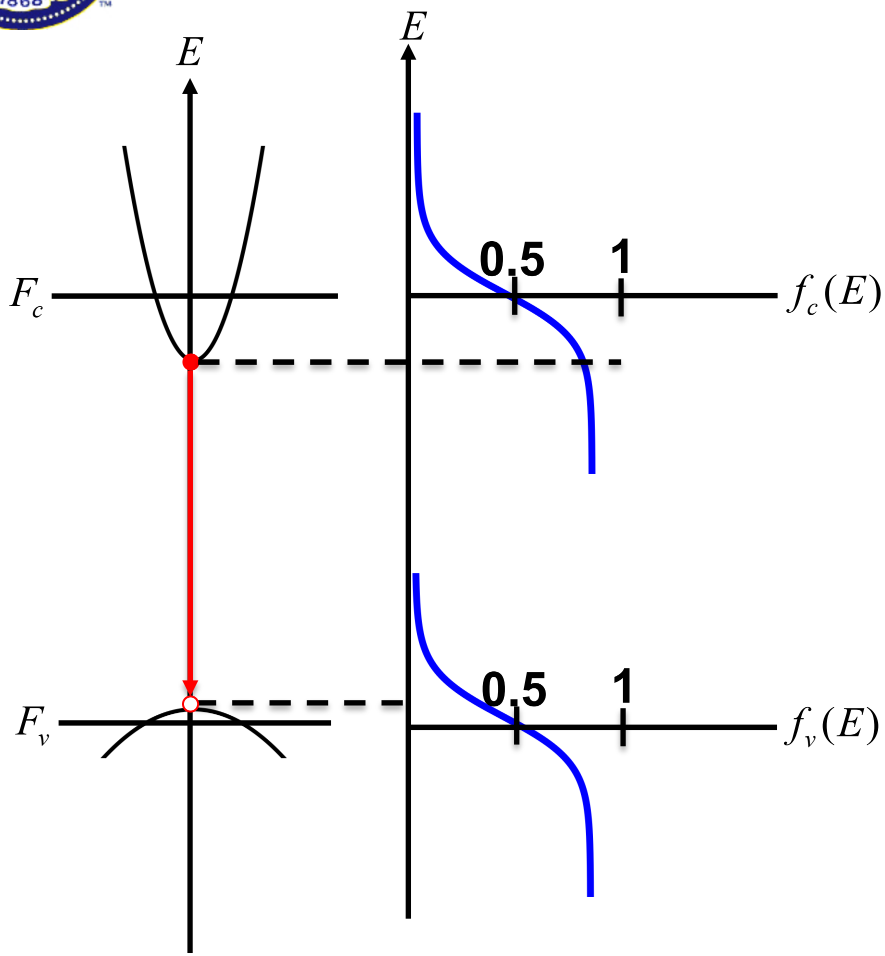
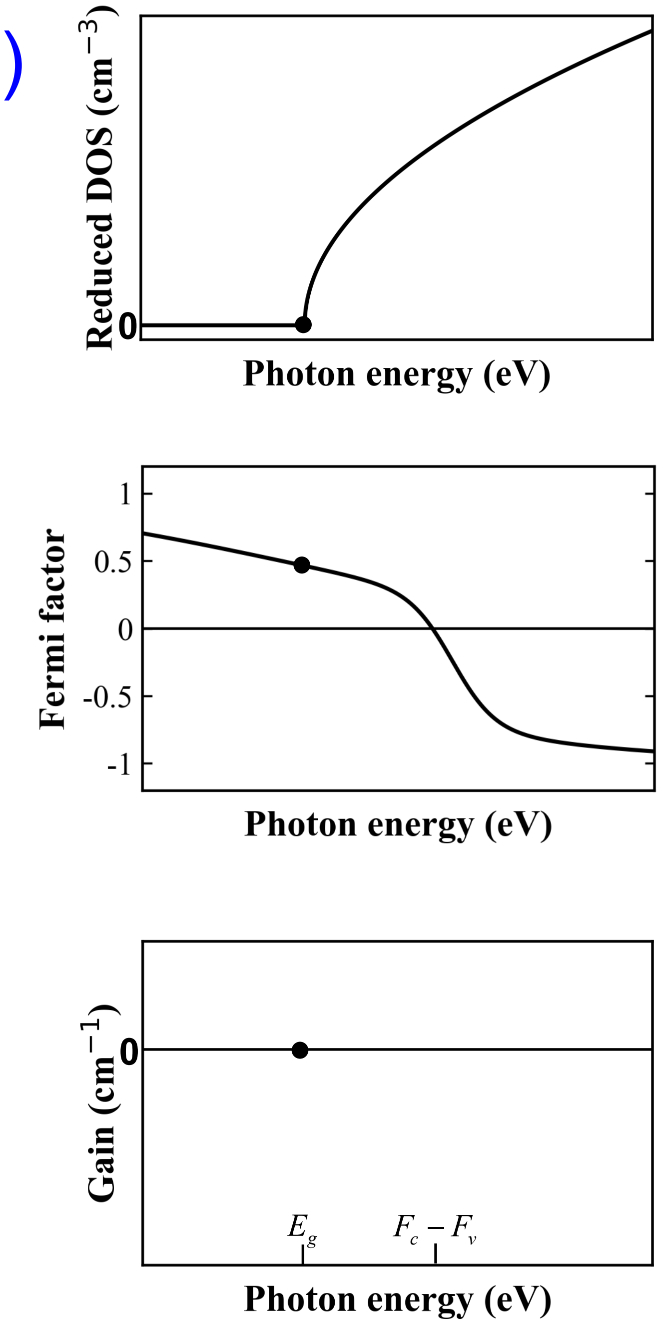
Fermi inversion factor

Gain spectrum (T=300K)





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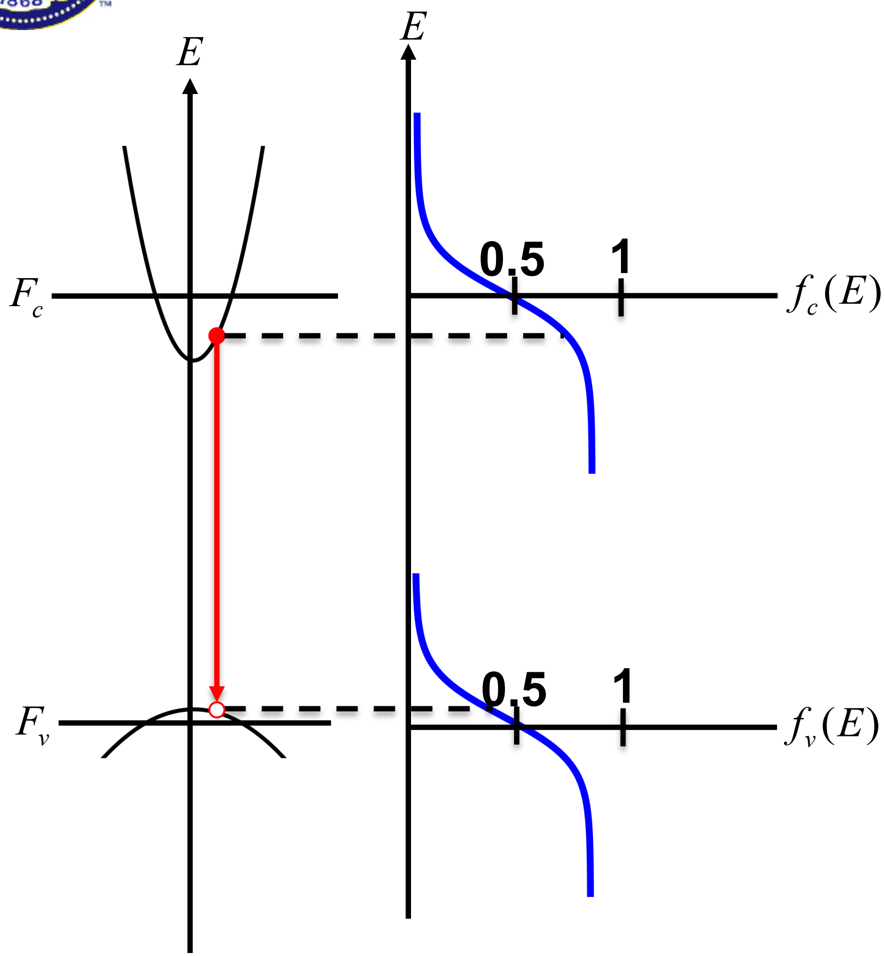

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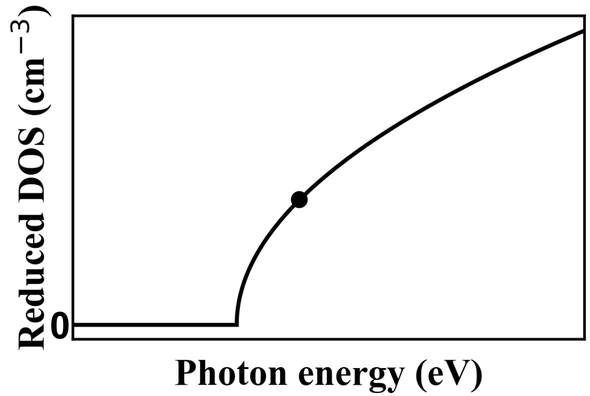

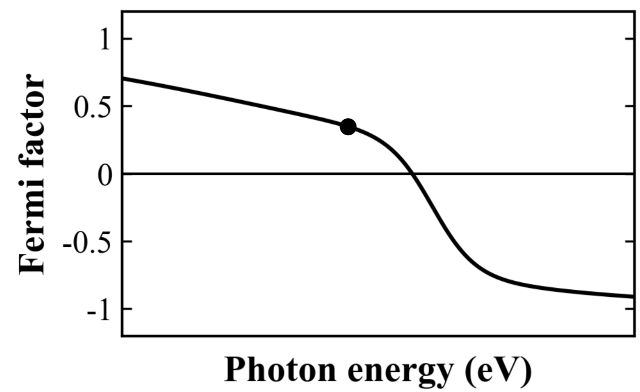

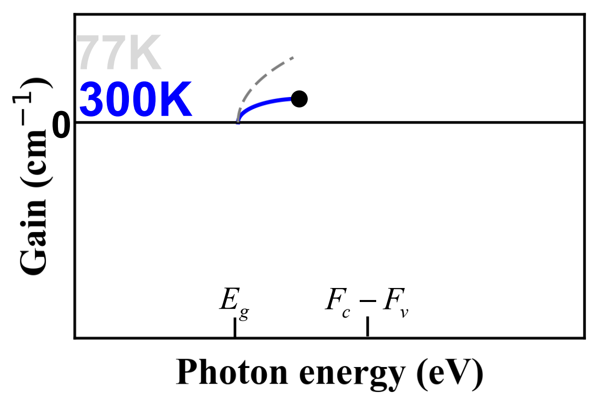

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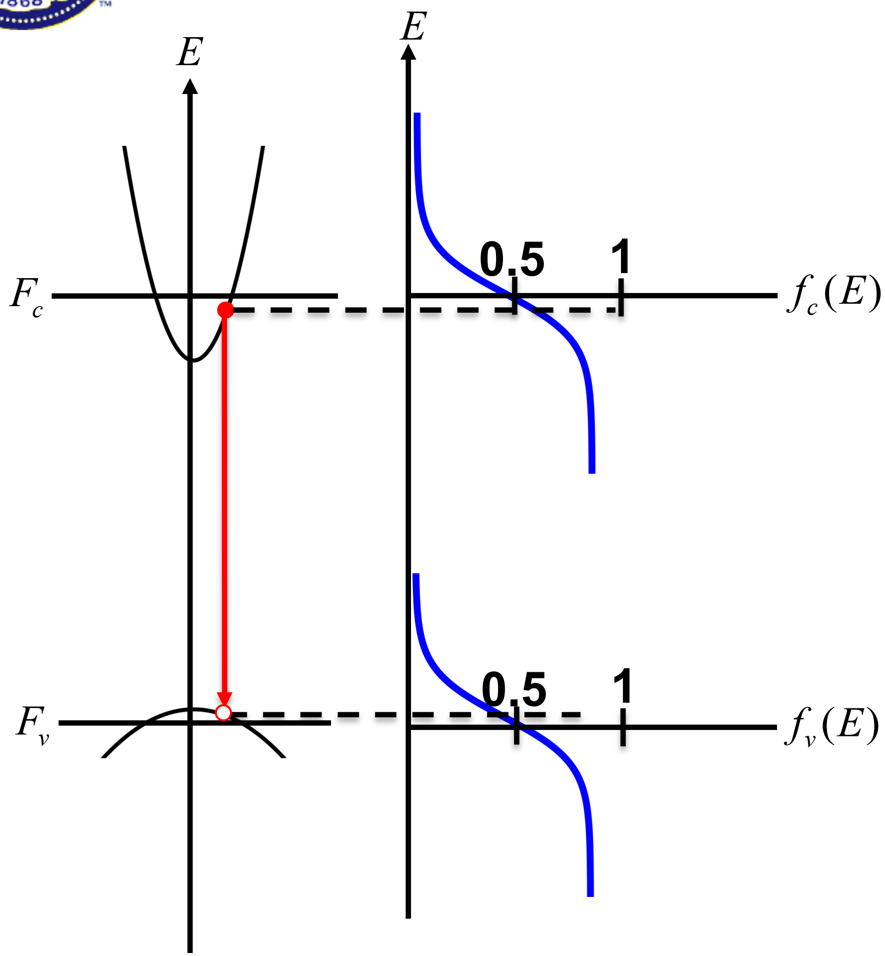

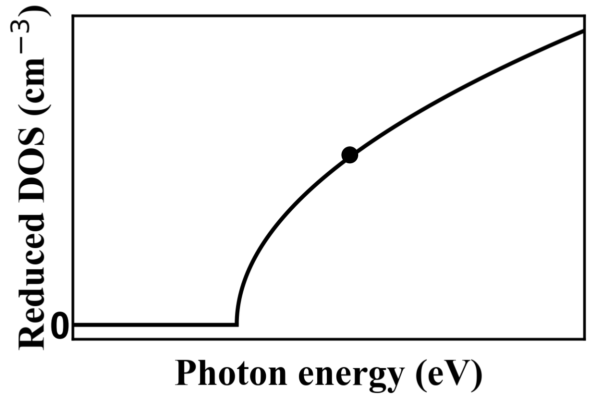

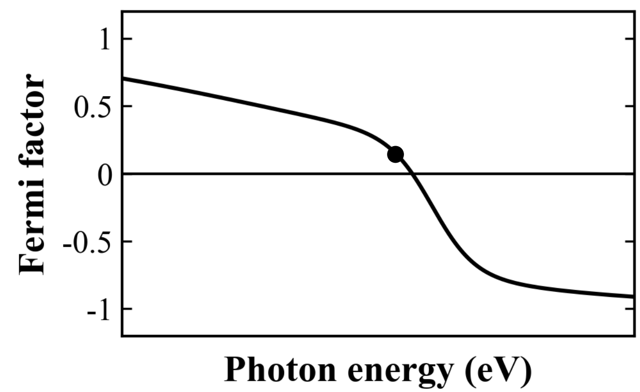

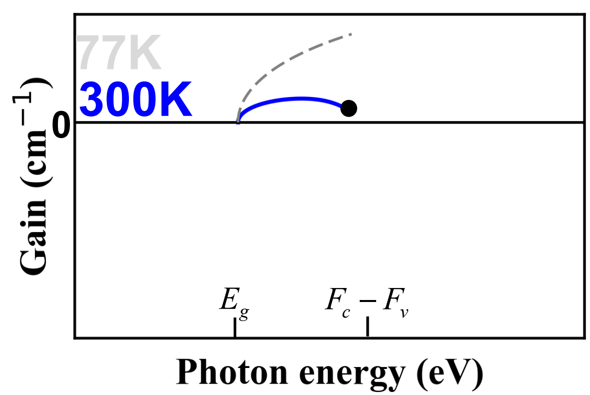

$$g(\hbar\omega) = -\alpha(\hbar\omega)$$

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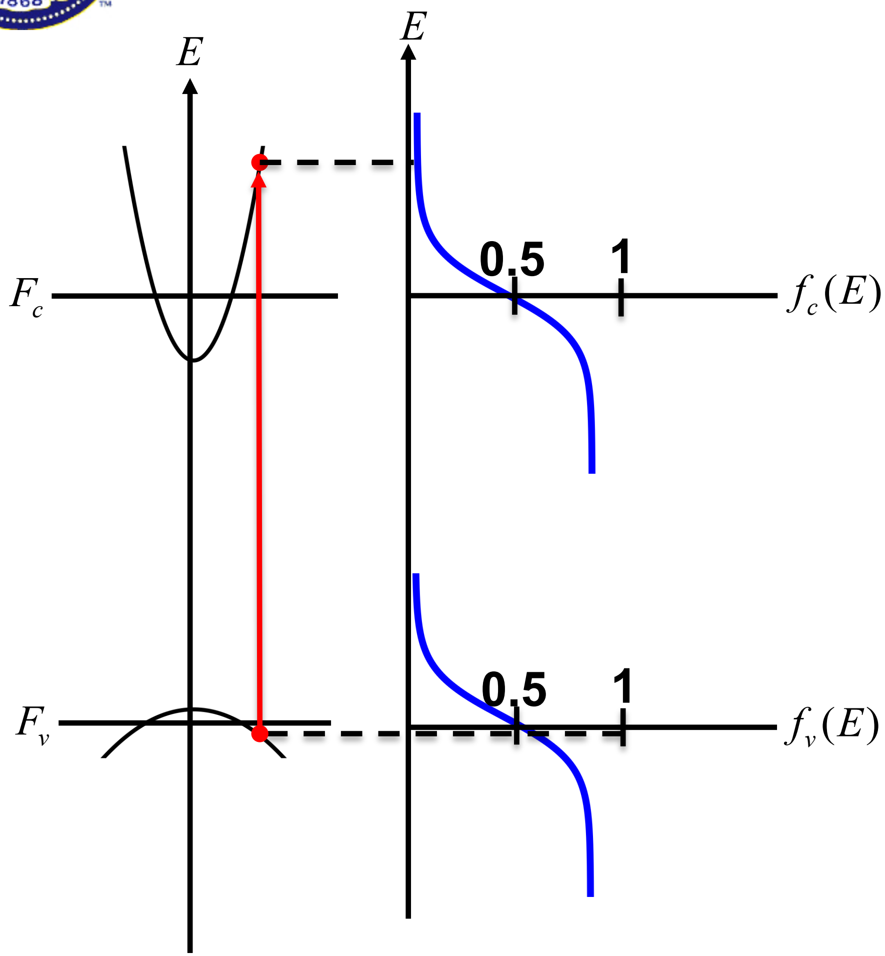

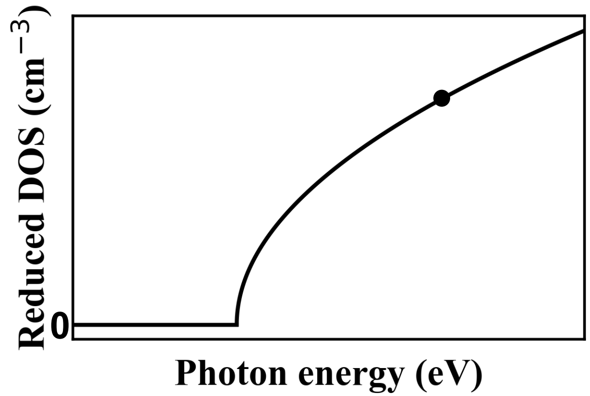

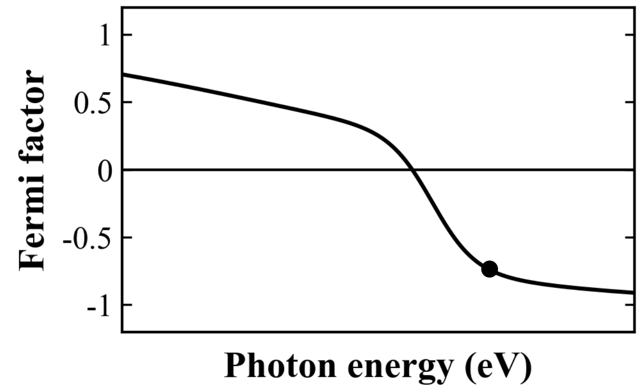

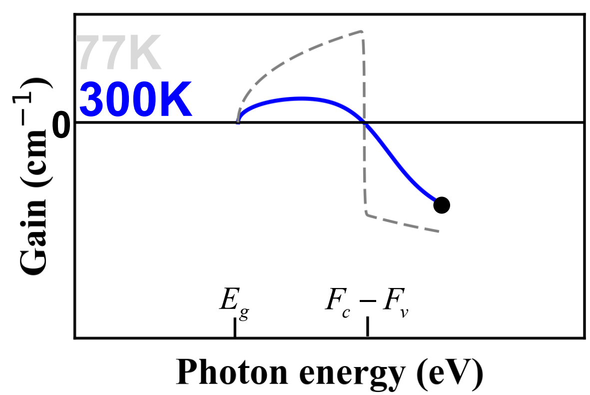

$$
\begin{aligned}
g(\hbar\omega) &= -\alpha(\hbar\omega) \\
&= C_0 M_b^2 \rho_r(\hbar\omega)\left[f_c(\hbar\omega) - f_v(\hbar\omega)\right]
\end{aligned}
$$

Fermi inversion factor

Gain spectrum

$$g(\hbar\omega) = -\alpha(\hbar\omega)$$
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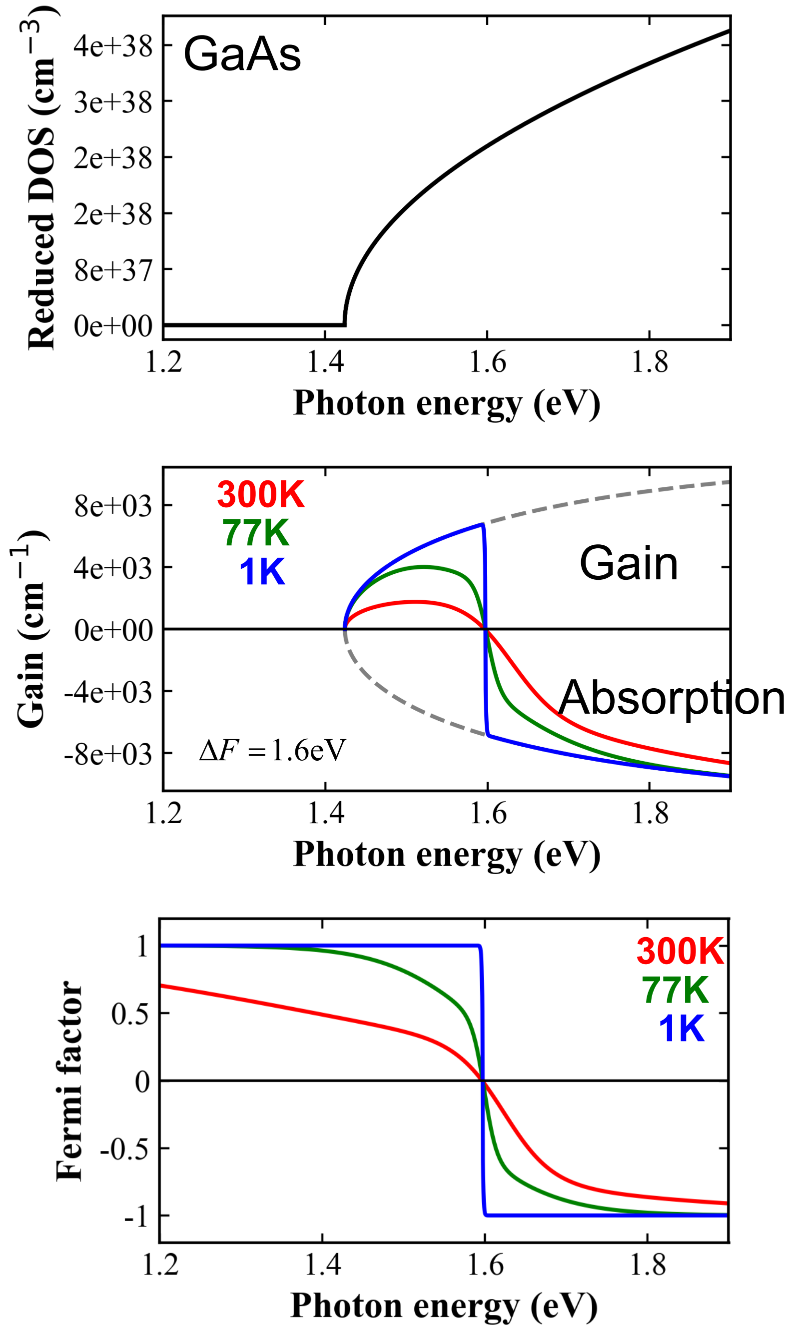
Fermi inversion factor



Calculated values for gallium arsenide (GaAs)

Gain spectrum

$$g(\hbar\omega) = -\alpha(\hbar\omega)$$
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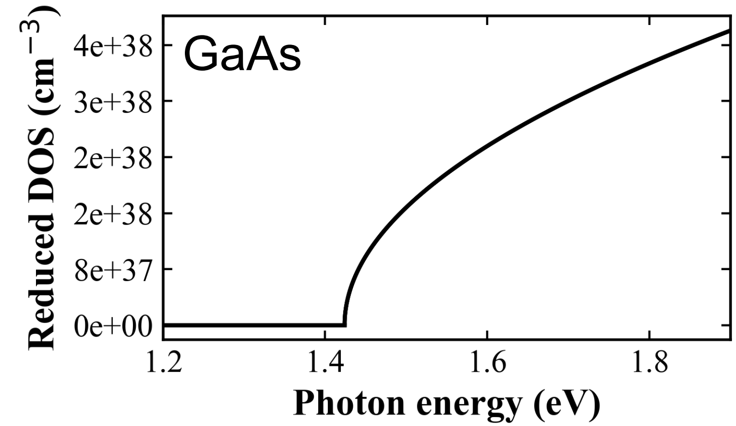
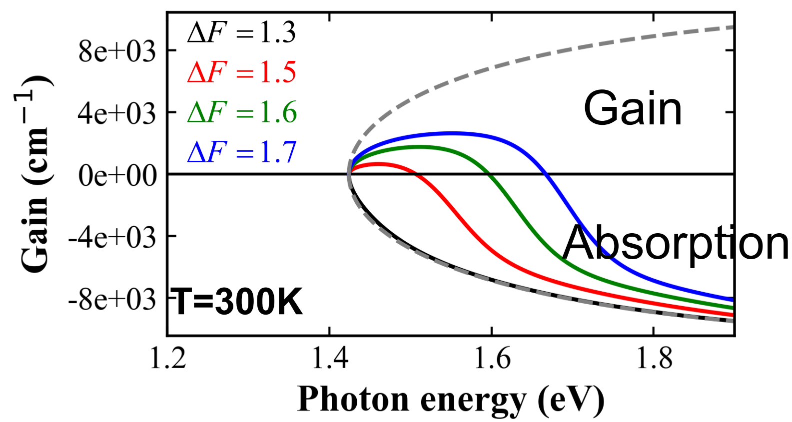
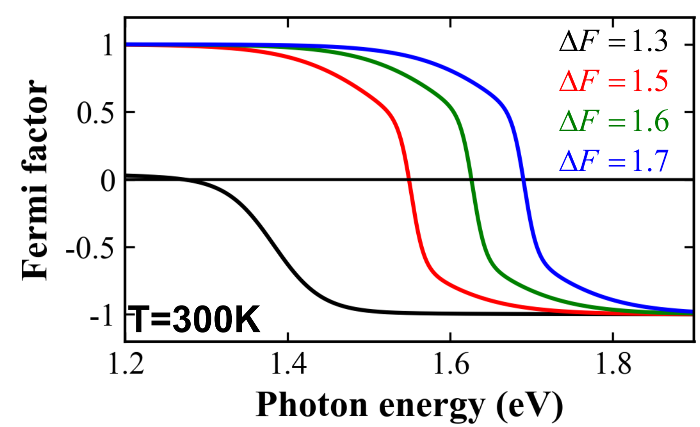
Fermi inversion factor

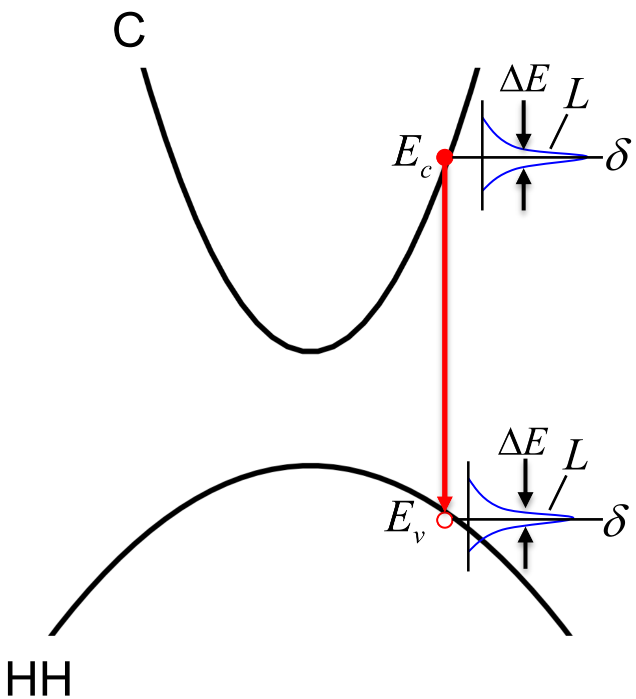




Calculated values for gallium arsenide (GaAs)



Linewidth broadening



- Electron and hole have a finite lifetime in a state due to various intraband scattering processes.

$$\delta(E_e - E_h - \hbar\omega) \rightarrow L(E_e - E_h - \hbar\omega, \tau_{in})$$

L: lineshape function

τ_{in}: intraband scattering time (~100 fs)

$$\mathcal{L}(E_e - E_h - \hbar\omega) = \frac{1}{\pi}\frac{\hbar\tau_{in}}{\left(\hbar/\tau_{in}\right)^2 + \left(E_e - E_h - \hbar\omega\right)^2}$$

Lorentzian lineshape if probability of finding an electron/hole in a given state decreases exponentially with time.

Linewidth broadening

$$\alpha = C_0 \frac{2}{V} \sum_{k_c} \sum_{k_v} |\hat{e} \cdot \mathbf{p}_{cv}|^2 \, \mathcal{L}(E_e - E_h - \hbar\omega)\left(f_v - f_c\right)$$

$$= C_0 |\hat{e} \cdot \mathbf{p}_{cv}|^2 \int \rho_r(E) \mathcal{L}(E_e - E_h - \hbar\omega)\left[f_v(E) - f_c(E)\right] dE$$

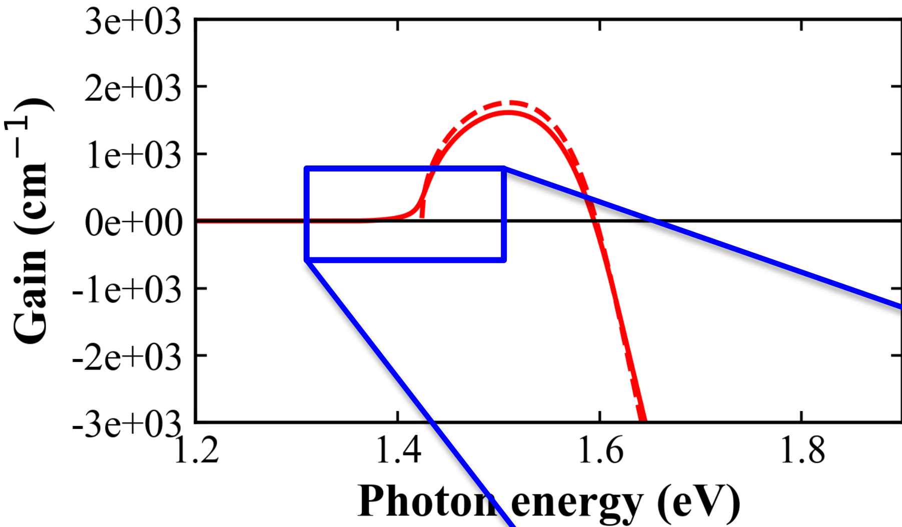
$$\boxed{\alpha = C_0 |\hat{e} \cdot \mathbf{p}_{cv}|^2 \int \rho_r(E) \mathcal{L}(E + E_g - \hbar\omega)\left[f_v(E) - f_c(E)\right] dE}$$

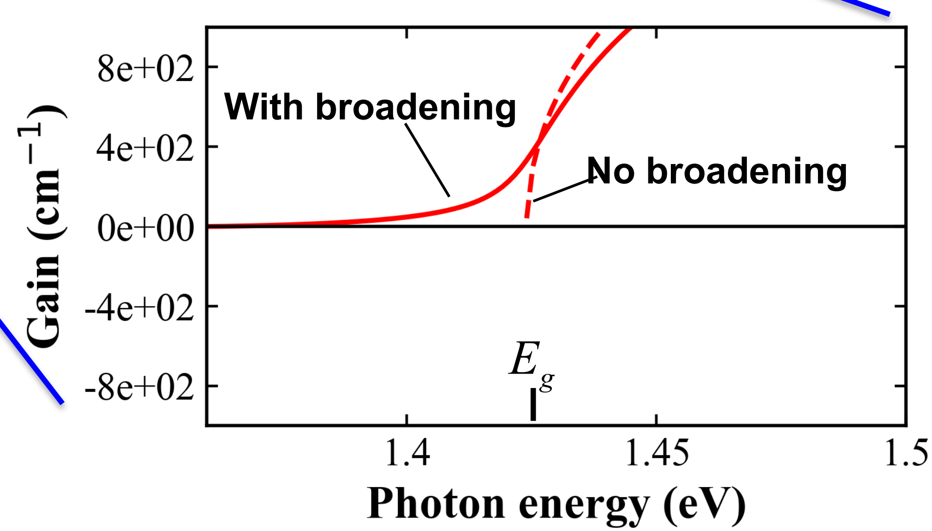
Recall $E_e - E_h = E_g + \dfrac{\hbar^2 k^2}{2m_r^*} = E + E_g$

$$f_c(E) = \frac{1}{1 + \exp[(E_g + E\, m_r^*/m_e^* - F_c)/kT]}$$

$$f_v(E) = \frac{1}{1 + \exp[(-E\, m_r^*/m_h^* - F_v)/kT]}$$

Linewidth broadening





- Lorentzian usually over-estimates sub-bandgap gain/absorption
- Will discuss other linewidth models later on.