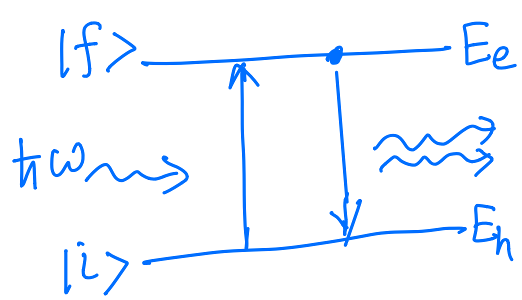
Midterm next Tuesday
Open book/note, closed internet
12:40 → 2:00. upload to bCourse.
Materials to last Tuesday, Microring Resonator

Active optoelectronic Materials

Fermi Golden Rules



$$W_{abs} = \frac{2\pi}{\hbar^2}|H'_{fi}|^2\,\delta(\omega_{fi} - \omega)$$

optical freq (ω)

difference electronic states energy (ω_{fi})

$E = \hbar\omega$

$\int f(x)\,\delta(x)\,dx$

Change of variable for $\delta(x)$

$$\delta(ax) = \frac{1}{a}\delta(x)$$

$$\delta(E) = \delta(\hbar\omega) = \frac{1}{\hbar}\delta(\omega)$$

$$W_{abs} = \frac{2\pi}{\hbar}|H'_{fi}|^2 \cdot \delta(E_e - E_h - \hbar\omega)$$

Stimulated emission

$$W_{emi} = \frac{2\pi}{\hbar}|H'_{fi}|^2 \cdot \delta(E_e - E_h - \hbar\omega)$$

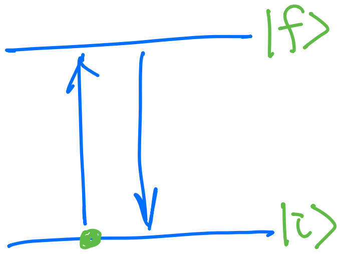
↳ Q: Why not $\delta(\hbar\omega - (E_e - E_h))$?

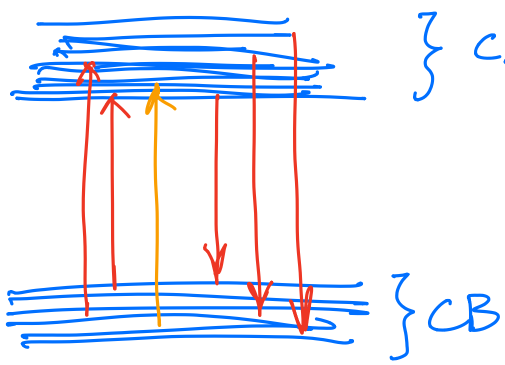
$\delta(-x) = \delta(x)$

2-level ⇒ Band in Semiconductor.

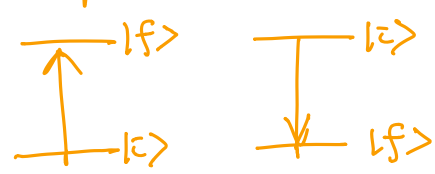
2 states ⇒ no Σ

Σ





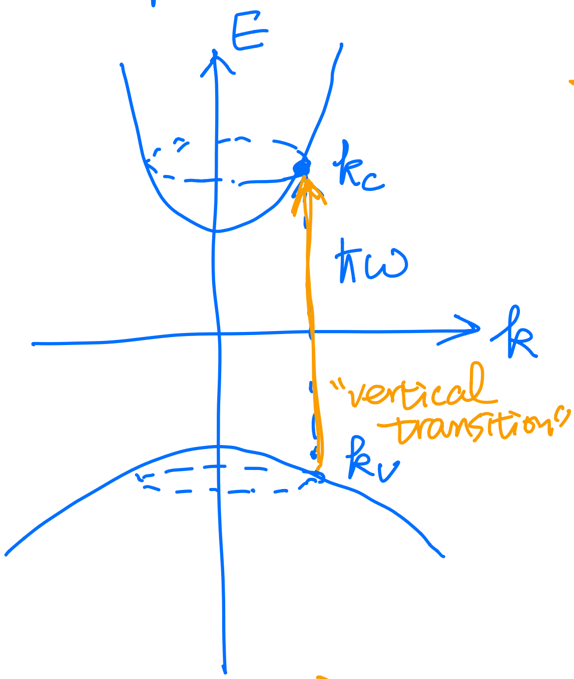
In previous lectures



Previous:

Initial condition: $C_i(t=0) = 1$, $C_f(t=0) = 0$

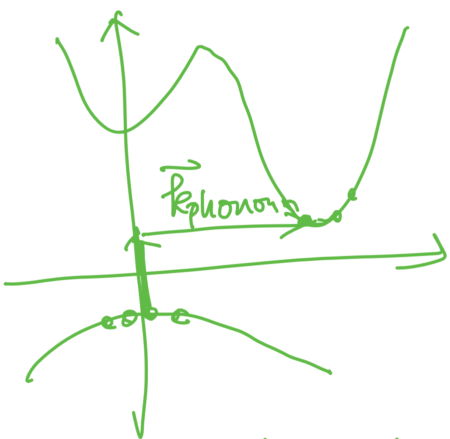
k-space



$$\vec{k}_c = \vec{k}_v + \vec{k}_{photon}$$

$$|\vec{k}_{photon}| \ll |\vec{k}_c|, |\vec{k}_v|$$

Q: Phonon-assisted optical transition.



Much weaker than Direct bandgap transition

$\Rightarrow$ Difficult to make lasers

$$\underline{R_{v\to c}} = \frac{2}{V}\sum_{\underline{k_c = k_v}} \frac{2\pi}{\hbar} |H'_{cv}|^2 \delta(E_e - E_h - \hbar\omega) f_v (1-f_c)$$

spin

volume

probability valence state is occupied

Probability conduction band state is empty

Total aborption rate from VB to CB for photon energy $= \hbar\omega$ per unit volume

$$R_{c\to v} = \frac{2}{V}\sum_{k_c = k_v} \frac{2\pi}{\hbar} |H'_{cv}|^2 \cdot \delta(E_e - E_h - \hbar\omega) f_c (1-f_v)$$

Net absorption rate per unit volume:

$$R = R_{v\to c} - R_{c\to v}$$

$$= \frac{2}{V}\sum_{k_c = k_v} \frac{2\pi}{\hbar} \underline{|H'_{cv}|^2} \cdot \delta(E_e - E_h - \hbar\omega)(f_v - f_c)$$

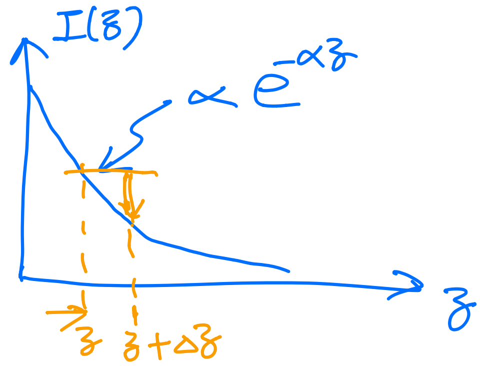
$\hookrightarrow \propto |E_{op}|^2$

No. of photons absorbed per unit volume per unit time

Experimentally measured is absorption coefficient

$$I(z) = I(0)\, e^{-\alpha z}$$

↑ optical intensity



$$\text{unit} = \left[\frac{J}{m^3}\cdot\frac{m}{s}\right] = \left[\frac{J}{m^2 \cdot s}\right]$$

$$I(z) = \frac{\epsilon E^2}{2}\cdot\frac{c}{n} = \frac{\epsilon_0 \epsilon_r E^2}{2}\,\frac{c}{n} = \frac{\epsilon_0 n^2 E^2}{2}\,\frac{c}{n} = \frac{\epsilon_0 n E^2 \cdot c}{2}$$

$$\epsilon_r = n^2$$

No. Photons aborbed per unit volume

$$R = \frac{I(z) - I(z+\Delta z)}{\Delta z}\cdot\frac{1}{\hbar\omega}$$

$$\left[\frac{J}{m^3 \cdot s}\right] \cdot \left[\frac{1}{J}\right] \rightarrow \left[\frac{1}{m^3 \cdot s}\right]$$

$$= \frac{-\frac{dI}{dz}\cdot\Delta z}{\Delta z}\cdot\frac{1}{\hbar\omega} = \frac{\alpha \cdot I\, \cancel{\Delta z}}{\cancel{\Delta z}}\,\frac{1}{\hbar\omega}$$

$$\alpha = \frac{R}{\left(\frac{I}{\hbar\omega}\right)} = \frac{R\cdot\hbar\omega}{I} = \frac{R\cdot 2\hbar\omega}{\epsilon_0 n E^2 c}$$

$$= \frac{2\cancel{\hbar}\omega}{\epsilon_0 n E^2 \cdot c}\;\frac{2}{V}\sum_{k_c=k_v}\left(\frac{2\pi}{\cancel{\hbar}}\right)\left|H'_{cv}\right|^2 \cdot \delta(E_e - E_h - \hbar\omega)\cdot(f_v - f_c)$$

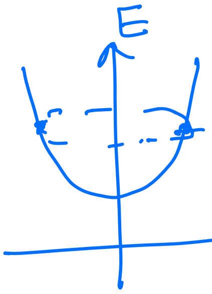
treat it as constant ∵ states are near bandedge

indep of k

Why

$$= \underbrace{\frac{4\pi\omega\left|H'_{cv}\right|^2}{\epsilon_0 n c E^2}}\underbrace{\left[\frac{2}{V}\sum_{k_c=k_v}\delta(E_e - E_h - \hbar\omega)\right]}_{\text{\# electron-hole } (k_c=k_v) \text{ states per unit volume}}\underbrace{(f_v - f_c)}$$

Earlier, density of states for conduction band:



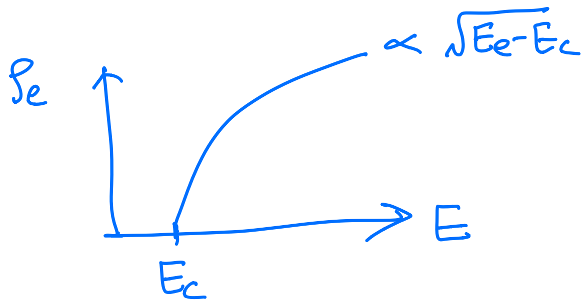
$E_e = E_c + \frac{\hbar^2 k^2}{2m_e^*}$

$$\frac{2}{V}\sum_k \delta(E - E_e) \to \int \rho(E)\cdot dE$$

$\uparrow$ electron density of states.

$$\boxed{E_e - E_c = \frac{\hbar^2 k^2}{2m_e^*}}$$

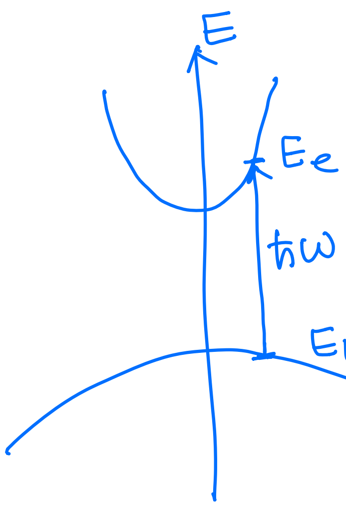
$\uparrow$ counting state



$$\rho_e(E_e) = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E_e - E_c}$$

Q: Is m_e^* constant?

Yes, under effective mass approximation



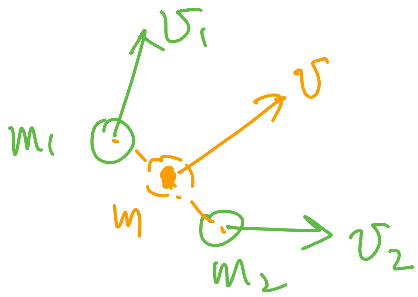
$$\left.\begin{aligned} E_e &= E_c + \frac{\hbar^2 k^2}{2m_e^*} \\ E_h &= E_v - \frac{\hbar^2 k^2}{2m_h^*} \end{aligned}\right\}$$

$$\underbrace{E_e - E_h}_{\hbar\omega} = \underbrace{(E_c - E_v)}_{E_g} + \frac{\hbar^2 k^2}{2}\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right)$$

"Reduced effective mass"

$$\frac{1}{m_r^*} = \frac{1}{m_e^*} + \frac{1}{m_h^*}$$

Mechanical Analogy



$$\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$\boxed{\hbar\omega - E_g = \frac{\hbar^2 k^2}{2m_r^*}}$$

"Reduced" Density of states

$$\rho_r(\hbar\omega) = \frac{1}{2\pi^2}\left(\frac{2m_r^*}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{\hbar\omega - E_g}$$

Also called

"Joint Optical" Density of States

$\uparrow$ electronic

$$\alpha = \frac{4\pi\omega |H'_{cv}|^2}{\epsilon_0 n c E^2} \cdot \rho_r(\hbar\omega) \cdot [f_v - f_c]$$

$|H'_{cv}|^2$

Dipole Approx:

$$H'_{cv} = \frac{\overbrace{q\vec{r}}^{\text{dipole moment}} \cdot \hat{e} E}{2}$$

$\hat{e}$: polarization (direction of $\vec{E}$)

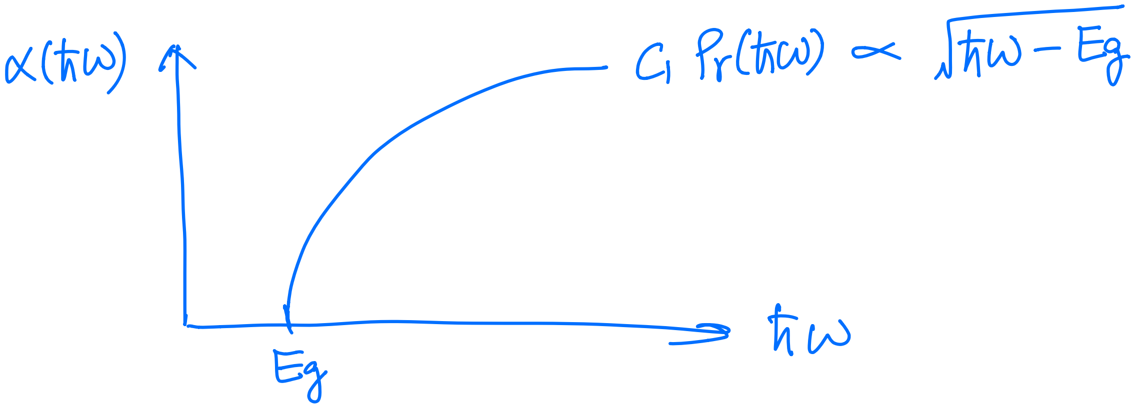
$$|H'_{cv}|^2 = \frac{q^2 E^2}{4} |\hat{e} \cdot \vec{r}_{cv}|^2$$

$$\vec{r}_{cv} = \underbrace{\int \psi_c^*(\vec{r}) \cdot \vec{r} \cdot \psi_v(\vec{r}) d\vec{r}}$$

$$\alpha = \underbrace{\left(\frac{\pi\omega q^2}{n c \epsilon_0}\right)}_{C_1} \rho_r(\hbar\omega) \cdot [f_v - f_c]$$

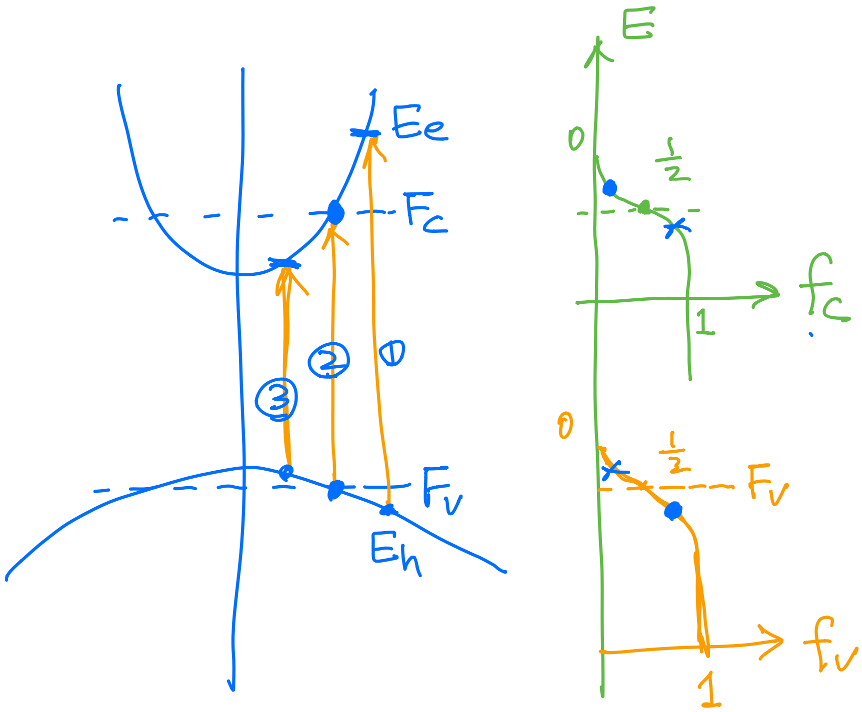
If semiconductor is undoped, unbiased

$f_v = 1$, $f_c = 0$ $\Rightarrow$ $f_v - f_c = 1$: maximum absorption



Semiconductor is biased (pumped)

$f_v - f_c \leq 1$



① $E_e > F_c$, $E_h < F_v$

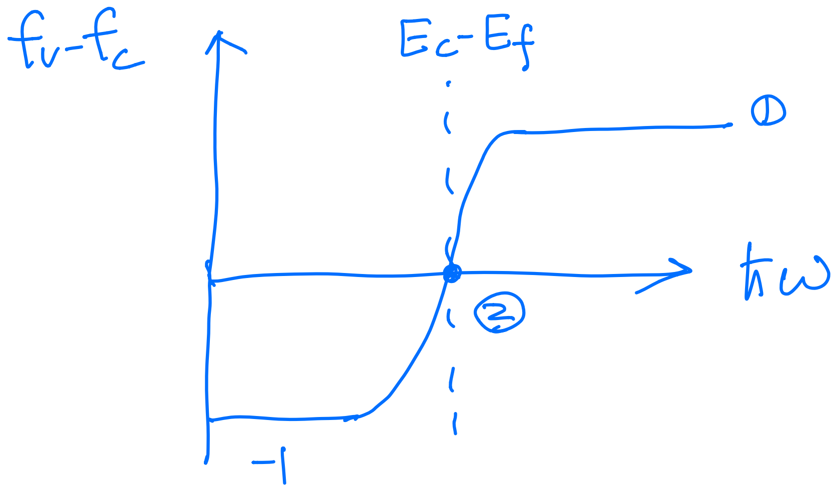
$f_v - f_c > 0$

② $E_e = F_c$, $E_h = F_v$

$f_v - f_c = \frac{1}{2} - \frac{1}{2} = 0$

③ $E_e < F_c$, $E_h > F_v$

$f_v - f_c < 0$



Fermi factor