Bulk Semiconductor

$$\alpha(\hbar\omega) = C_0 M_b^2 \, \rho_r(\hbar\omega)\, [\underbrace{f_v - f_c}]$$

Matrix element

Fermi Factor
only a function of $\hbar\omega$, F_c, F_v

$f_v - f_c > 0$: Absorption
$f_v - f_c = 0$: Transparent (for $\hbar\omega$)
$f_v - f_c < 0$: Gain $\rightarrow$ Stimulated emission

$$f_c = \frac{1}{1 + e^{(E_e - F_c)/k_BT}} \; ; \; f_v = \frac{1}{1 + e^{(E_h - F_v)/k_BT}}$$

Transparency Cond

$f_c = f_v \Rightarrow E_e - F_c = E_h - F_v$

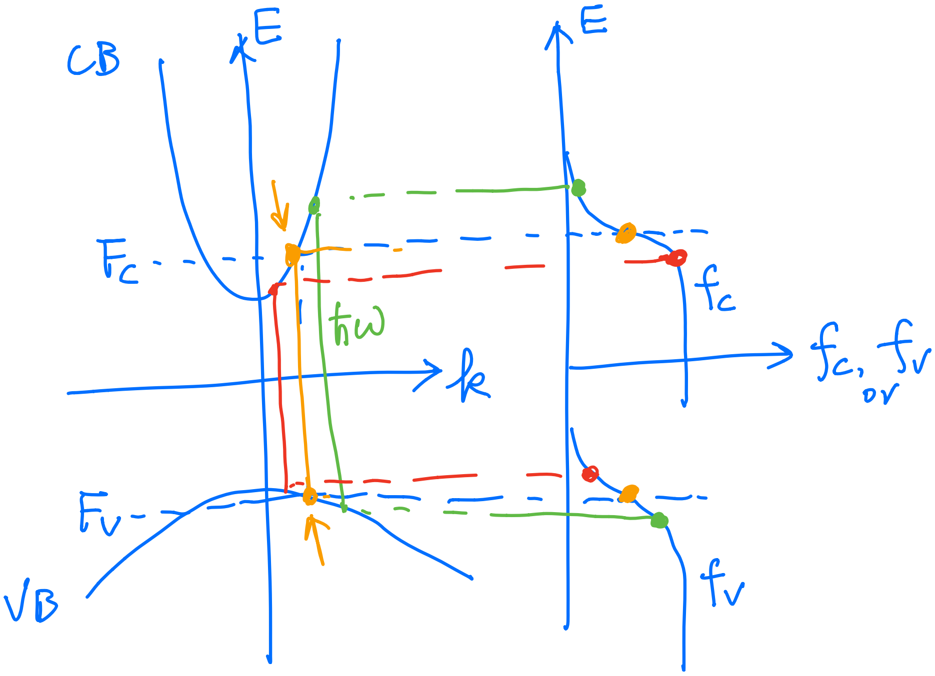
$\Rightarrow (E_e - E_h) = (F_c - F_v) = \Delta F$

$\Rightarrow \hbar\omega = \Delta F$

$= q \cdot V_{applied}$ of p-n

Bernard-Durofoug Condition
↳ 1961

$\Delta F > \hbar\omega \Rightarrow$ Gain
$\Delta F < \hbar\omega \Rightarrow$ Absorption
Unbiased semiconductor. $\Delta F = 0 < \hbar\omega$



$\hbar\omega > \Delta F_c$, $f_c < \frac{1}{2}$, $f_v > \frac{1}{2}$. $f_v - f_c > 0$ Absorption

$\hbar\omega = \Delta F_c$ $f_c = \frac{1}{2}$, $f_v = \frac{1}{2}$ $f_v - f_c = 0$ Transparency

$\hbar\omega < \Delta F_c$ $f_c > \frac{1}{2}$. $f_v < \frac{1}{2}$ $f_v - f_c < 0$ Gain

Under a fixed ΔF , $\Delta F > E_g$
Spectral width of gain

$$E_g < \hbar\omega < \Delta F$$

$$n = N_c \frac{4}{3\sqrt{\pi}} \cdot \left(\frac{F_c - E_c}{k_B T}\right)^{3/2}$$

$\rightarrow \propto m_e^{*\,3/2}$

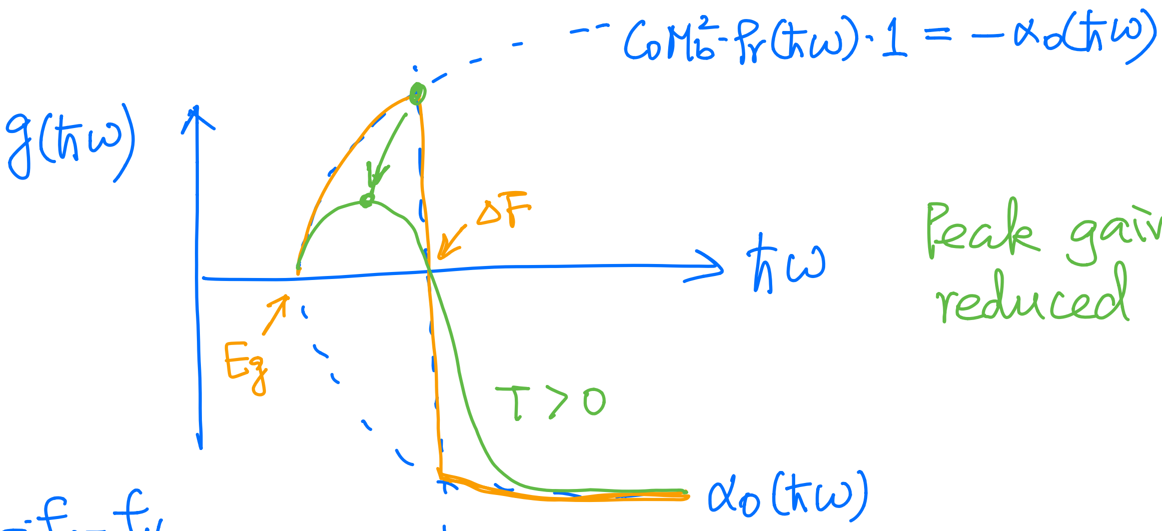
$$F_c - E_c \propto \frac{1}{m_e^*}$$

$$E_e = E_c + \frac{\hbar^2 k^2}{2m_e^*} \Rightarrow E_e - E_c \propto \frac{1}{m_e^*}$$

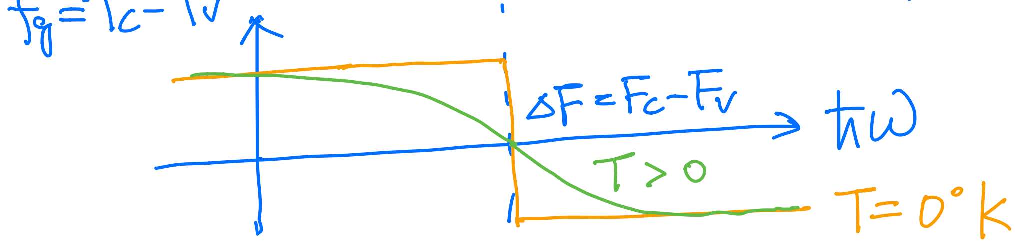
Gain [cm^{-1}] or [m^{-1}]

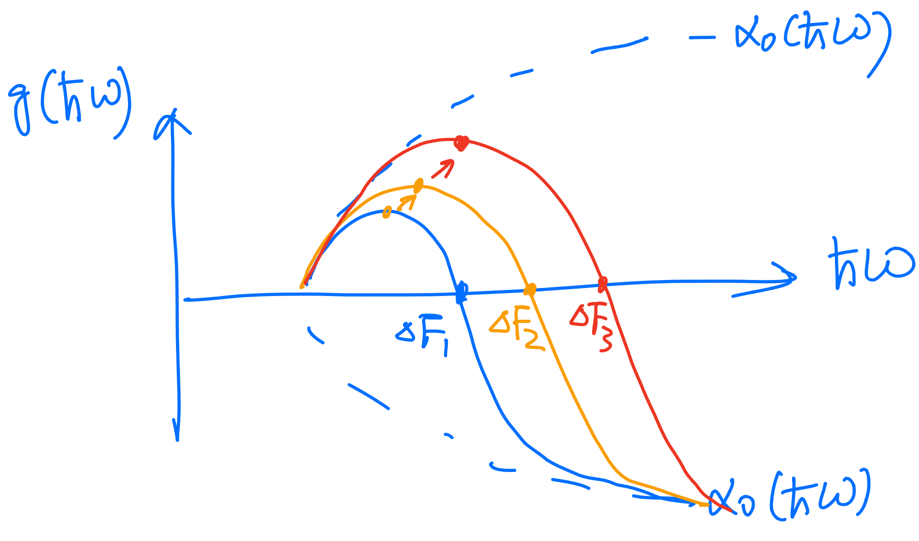
$$g(\hbar\omega) = -\alpha(\hbar\omega) = C_0 M_b^2 \underbrace{\rho_r(\hbar\omega)}_{\propto \sqrt{\hbar\omega - E_g}} \underbrace{[f_c - f_v]}_{\text{Fermi Inversion Factor}}$$

C_0: weak dep on ω



Peak gain is reduced at $T > 0°K$

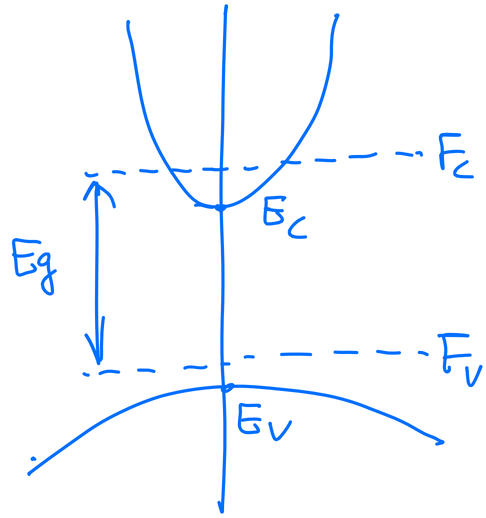




$\Delta F = \hbar\omega$ Transparency

Transparency carrier conc. n_{tr}

$\hbar\omega = E_g = \Delta F$, $n = p = n_{tr}$



$n > N_c$ $\quad n = N_c \cdot \frac{4}{3\sqrt{\pi}} \left(\frac{F_c - E_c}{k_B T}\right)^{3/2}$

$$\Rightarrow F_c = \left(\frac{n_{tr}}{N_c} \frac{3\sqrt{\pi}}{4}\right)^{2/3} \cdot k_B T + E_c \cdots ①$$

$p < N_v$, $p = N_v e^{-\frac{F_v - E_v}{k_B T}}$

$$\Rightarrow F_v = E_v + k_B T \cdot \ln\left(\frac{n_{tr}}{N_v}\right) - ②$$

$m_e^* \ll m_h^*$

GaAs

$0.067\, m_0 \ll 0.45\, m_0^*$

① - ②

$$\underbrace{F_c - F_v}_{E_g} = \underbrace{E_c - E_v}_{E_g} + \left(\frac{n_{tr}}{N_c} \cdot \frac{3\sqrt{\pi}}{4}\right)^{2/3} k_B T - k_B T \cdot \ln\left(\frac{n_{tr}}{N_v}\right)$$

$$\left(\frac{n_{tr}}{N_c} \cdot \frac{3\sqrt{\pi}}{4}\right)^{2/3} = \ln\left(\frac{n_{tr}}{N_v}\right)$$

$n_{tr} \sim 10^{18}\ cm^{-3}$

Non-degerate

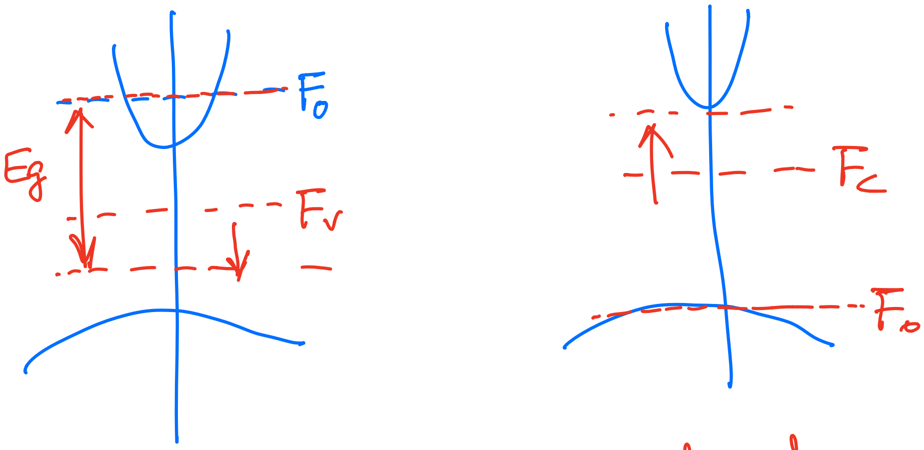
$$\begin{cases} n \simeq N_c \cdot e^{-\frac{E_c - F}{k_B T}} \\ p \simeq N_v\, e^{+\frac{E_v - F}{k_B T}} \end{cases}$$

↑ "Lumped" density of states.
of. CB, VB

Undoped semiconductor , $n = N_D + \Delta n$

↑ bias

$p = N_A + \Delta n$

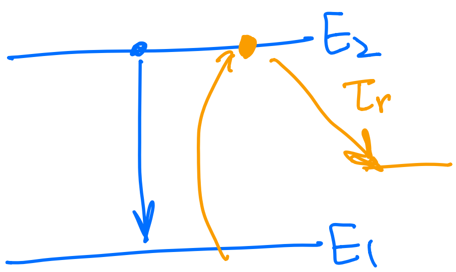


N-doped   P-doped

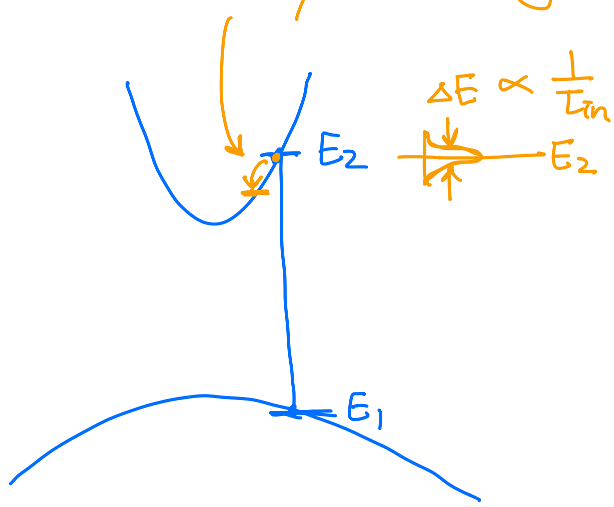
Q: If we want to reduce η_{tr}, does p-doping or n-doping more effective ?

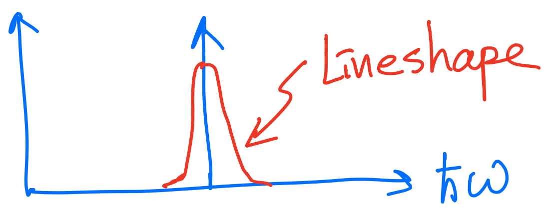
Lineshape function

2-level system



Intra-valley scattering time : τ_{in}





$\tau_{in} \sim 100$ fs

$$g(\hbar\omega) = C_0 M_b^2 \cdot \rho_r(\hbar\omega) \cdot f_g$$

$$\int \rho_r(E) \cdot \delta(E - \hbar\omega)\, dE$$

Uncertainty Principle

$$\Delta E \cdot \Delta t > \frac{1}{2}\hbar$$

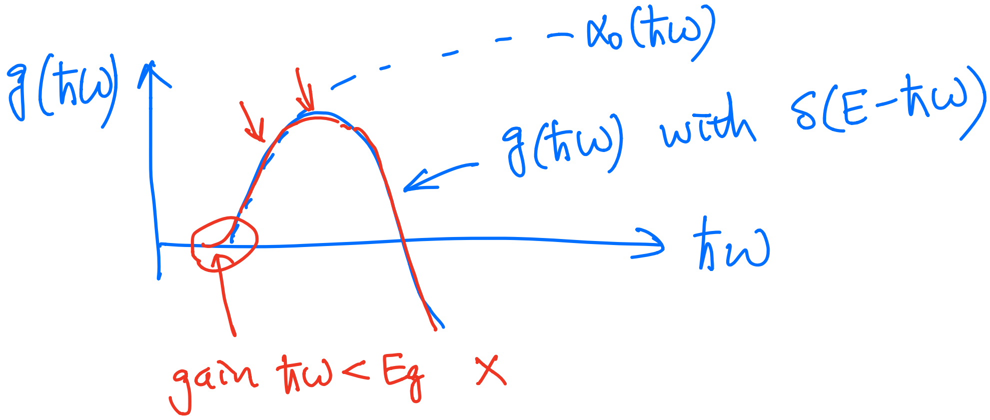
measurement time

Replace

$$\delta(E - \hbar\omega) \rightarrow \mathcal{L}(E - \hbar\omega)$$

Lineshape function

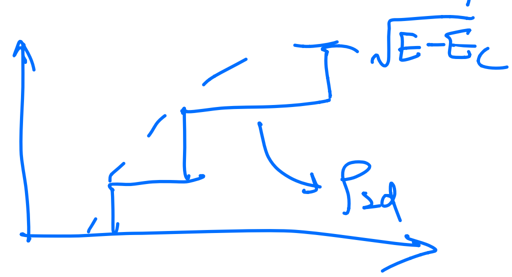
$$\mathcal{L}(E-\hbar\omega) = \frac{1}{\pi}\frac{\hbar\tau_{in}}{\left(\frac{\hbar}{\tau_{in}}\right)^2 + (E-\hbar\omega)^2}$$



Next Time:

Absorption/Gain in Quantum Wells.

Review 2D Density of States



Hole bands
- Heavy hole
- Light hole

Email me, Piazza, for OH 2-3