EE 232 Lightwave Devices
Lecture 20: Steady State Response of Semiconductor LEDs and Lasers

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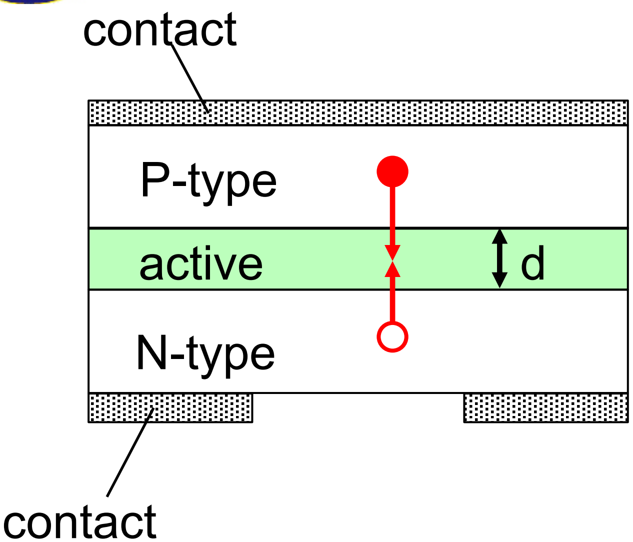
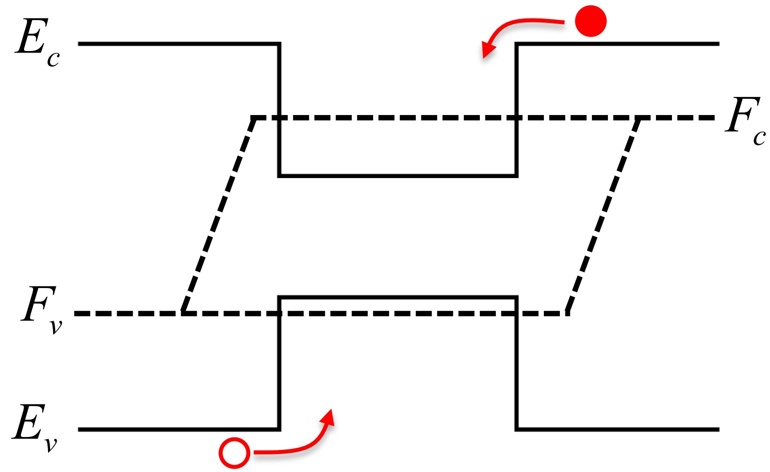
Light emitting diode at steady state





$$\frac{\partial n}{\partial t} = -\nabla \cdot \mathbf{J} - R$$

Rate equation for active region (ignoring generation term)

We neglect current spreading in the lateral direction and assume quasi-Fermi level is constant throughout active region.

$$\frac{dn}{dt} \simeq \eta_i \frac{J}{qd} - R$$

η_i: injection efficiency; R: carrier recombination

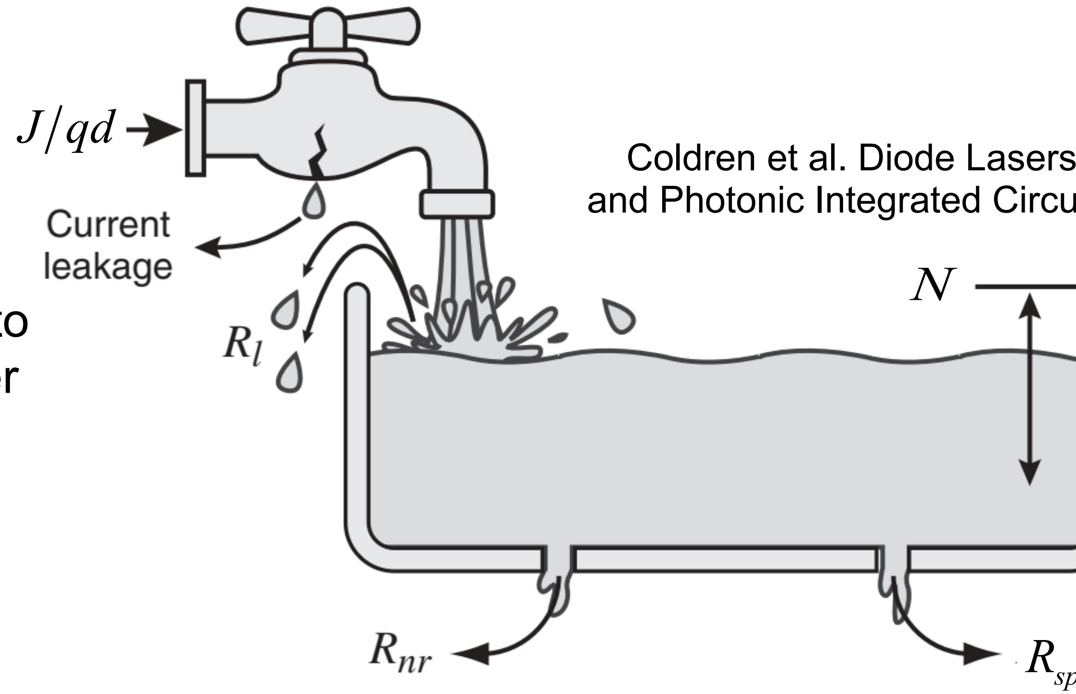
At steady state: $\eta_i \frac{J}{qd} = R$

d is active region thickness

Carrier recombination

Leaky Bathtub analogy

At steady state,
we must continuously pump the active region to maintain a certain carrier density



Coldren et al. Diode Lasers and Photonic Integrated Circuits.

$$R = R_{SRH} + R_{sp} + \cancel{R_{stim}} + R_{Auger}$$

- R_{SRH}: Shockley-Reed-Hall recombination
- R_{sp}: Spontaneous emission
- R_{stim}: Stimulated emission
- R_{Auger}: Auger recombination

Spontaneous emission power

$$\eta_i \frac{J}{qd} = R_{SRH} + R_{sp} + R_{Auger}$$

$$\text{Let } \eta_{sp} = \frac{R_{sp}}{R_{SRH} + R_{sp} + R_{Auger}} \quad \Rightarrow \quad R_{sp} = \eta_{sp}\eta_i \frac{J}{qd}$$

Spontaneous emission power (P_{sp})

$$= (\text{Photon energy}) \times (\text{Emission rate}) \times (\text{Active region volume})$$

$$\begin{aligned} P_{sp} &= \hbar\omega R_{sp} V_{act} \\ &= \hbar\omega V \eta_{sp}\eta_i J / qd \\ &= \eta_{sp}\eta_i \frac{\hbar\omega}{q} I \end{aligned}$$

$$\boxed{P_{sp} = \eta_{IQE} \frac{\hbar\omega}{q} I}$$

For constant internal quantum efficiency (IQE), there is a linear relationship between power and current. However, in general, IQE does change with current

ABC approximation

$$\eta_i \frac{J}{qd} = R_{SRH} + R_{sp} + R_{Auger}$$

$$\simeq An + Bn^2 + Cn^3$$

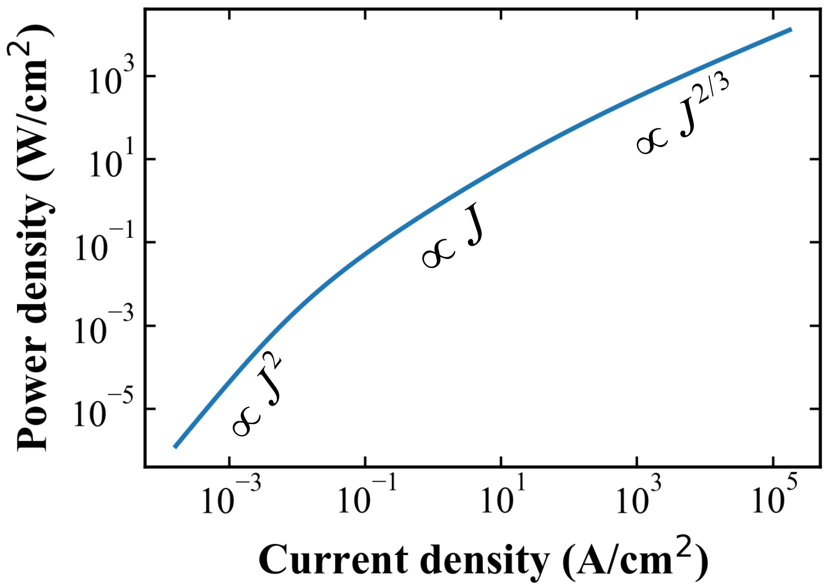
$$\eta_{IQE} = \eta_i \eta_{sp} = \eta_i \frac{R_{sp}}{R_{SRH} + R_{sp} + R_{Auger}} = \eta_i \frac{Bn^2}{An + Bn^2 + Cn^3}$$

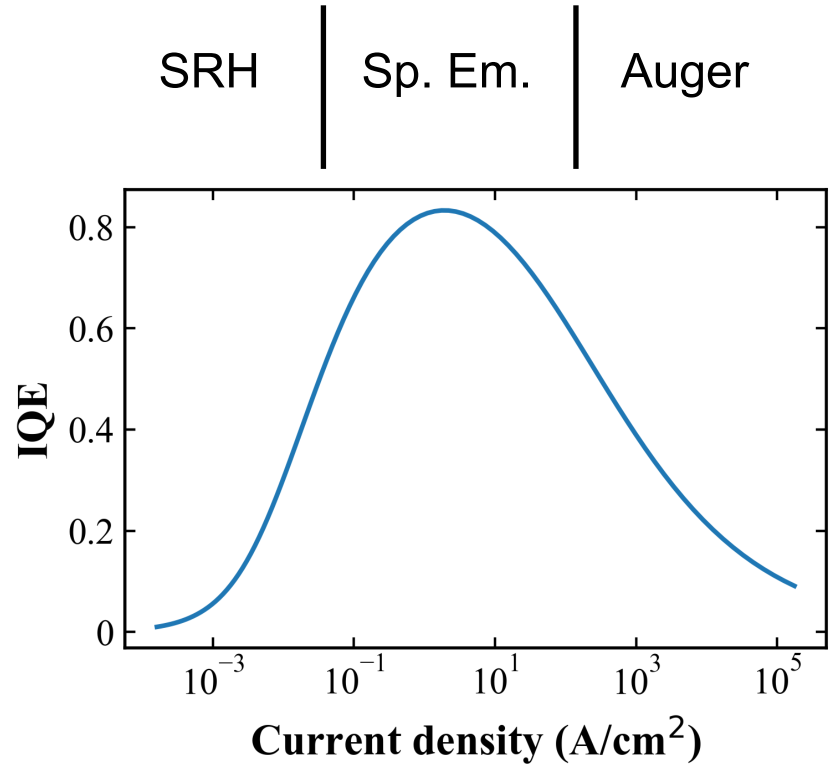
$$\eta_{IQE} = \eta_i \frac{Bn}{A + Bn + Cn^2}$$

Low drive current	Moderate drive current	High drive current
$A > Bn + Cn^2$	$Bn > A + Cn^2$	$Cn^2 > A + Bn$
$n \propto J$	$n \propto J^{1/2}$	$n \propto J^{1/3}$
$\eta_{IQE} \propto J$	$\eta_{IQE} \sim \text{constant}$	$\eta_{IQE} \sim J^{-1/3}$
$P_{sp} \propto J^2$	$P_{sp} \propto J$	$P_{sp} \propto J^{2/3}$

ABC approximation

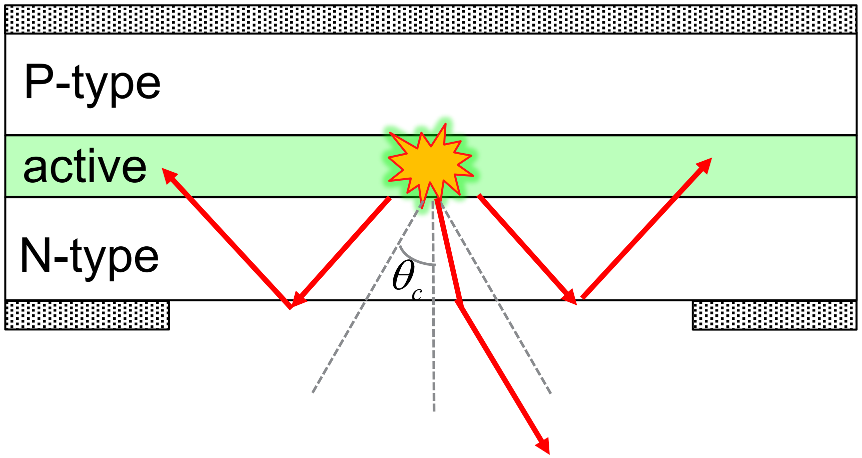
Ex. InGaAs LED





Note: At high current density, other loss mechanisms beside Auger may become important. See for example, controversy surrounding “droop” in GaN LED efficiency at high current.

LED external quantum efficiency



Most emitted light will become trapped in the high-index semiconductor. Only light with angle smaller than critical angle can escape.

$$\sin\theta_c = \frac{n_{air}}{n} \rightarrow \theta_c \sim \frac{1}{n}$$

Light extraction efficiency

$$\eta_c = \frac{\int T(\theta)\,d\Omega}{\int d\Omega} = \frac{1}{4\pi}\int_0^{\theta_c} 2\pi\sin\theta\,d\theta$$

$$\eta_c = \frac{1}{2}\cos\theta\Big|_0^{\theta_c} = \frac{1}{2}\left(1-\sqrt{1-\left(\frac{1}{n}\right)^2}\right)$$

$$\eta_c \approx \frac{1}{2}\left(1-\left(1-\frac{1}{2n^2}\right)\right) = \frac{1}{4n^2}$$

$$\eta_c \approx 2\% \quad \text{for } n \sim 3.5$$

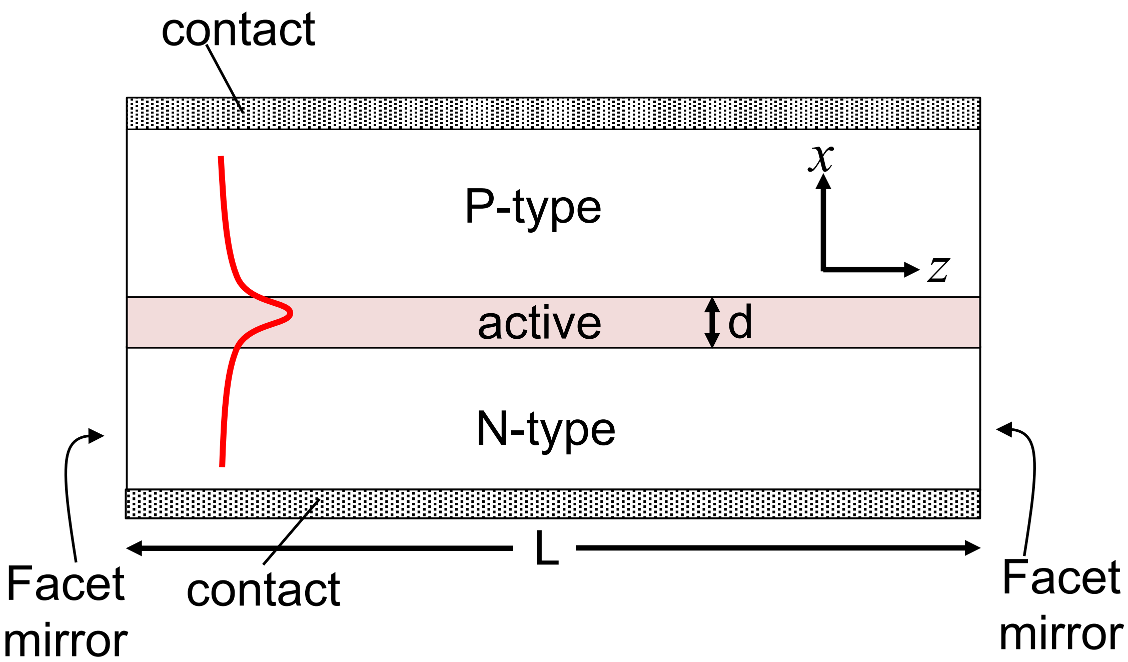
Externally collected power

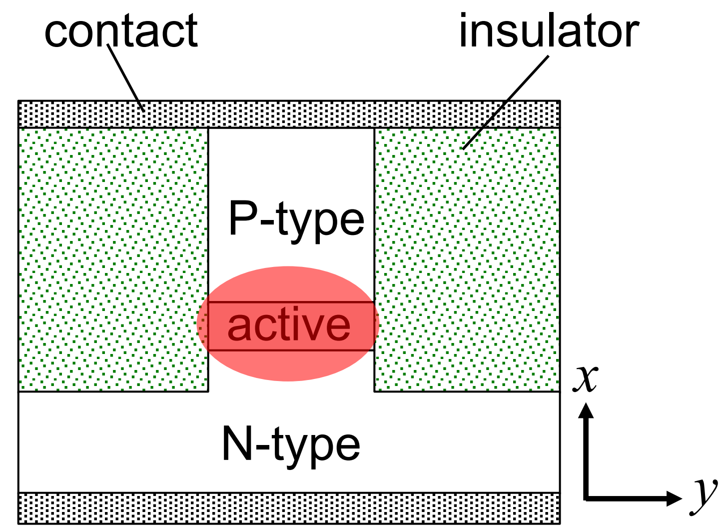
$$P_{sp} = \eta_{IQE}\eta_c\frac{\hbar\omega}{q}I = \eta_{ext}\frac{\hbar\omega}{q}I$$

External quantum efficiency

Light extraction efficiency can be increased by roughening the surface or adding nanostructures

Semiconductor laser at steady state





Edge-emitting ridge laser

$S \equiv$ photon density in lasing mode

$n \equiv$ carrier density in active region

$v_g \equiv$ group velocity

$\beta_{sp} \equiv$ fraction of sp. em. into lasing mode

$\tau_p \equiv$ photon lifetime

$g \equiv$ gain

$V_{cav} \equiv$ cavity volume

$V_{act} \equiv$ active region volume

$$\Gamma \simeq \frac{V_{act}}{V_{cav}} = \frac{(\text{Area})d}{V_{cav}} \rightarrow V_{cav} = \frac{(\text{Area})d}{\Gamma}$$

Rate equations for photons and carriers

Rate equation for carriers in active region

$$\frac{dn}{dt} = \eta_i \frac{J}{qd} - R_{SRH} - R_{sp} - R_{stim} - R_{Auger}$$

- R_{SRH}: Shockley-Reed-Hall recombination
- R_{sp}: Spontaneous emission
- R_{stim}: Stimulated emission
- R_{Auger}: Auger recombination

Rate equation for photons in cavity

$$\frac{dS}{dt} = -\frac{S}{\tau_p} + \Gamma \beta_{sp} R_{sp} + \Gamma R_{stim}$$

- $\frac{S}{\tau_p}$: Rate of photons leaving cavity
- $\Gamma \beta_{sp} R_{sp}$: Sp. em. rate into cavity mode
- ΓR_{stim}: Stimulated emission

Simplified Steady State Analysis

Ignore spontaneous emission: $R_{sp} \approx 0$

$$\frac{dS}{dt} = -\frac{S}{\tau_p} + \Gamma v_g g S = 0$$

Above threshold, $S > 0$

$$\Gamma v_g g = \frac{1}{\tau_p}$$

$$g = \frac{1}{\Gamma v_g \tau_p} = \frac{\alpha_i + \alpha_m}{\Gamma} = g_{th}$$

Gain "clamped" at threshold gain when above threshold

$\Rightarrow$ Carrier concentration is also clamped at threshold value

Simplified Steady State Analysis

$$\frac{dn}{dt} = \eta_i \frac{J}{qd} - R_{SRH} - \Gamma v_g g S - R_{Auger} = 0$$

(1) Below threshold: $S = 0$

$$J = \frac{qd}{\eta_i}\left(R_{SRH} + R_{Auger}\right)$$

(2) Above threshold:

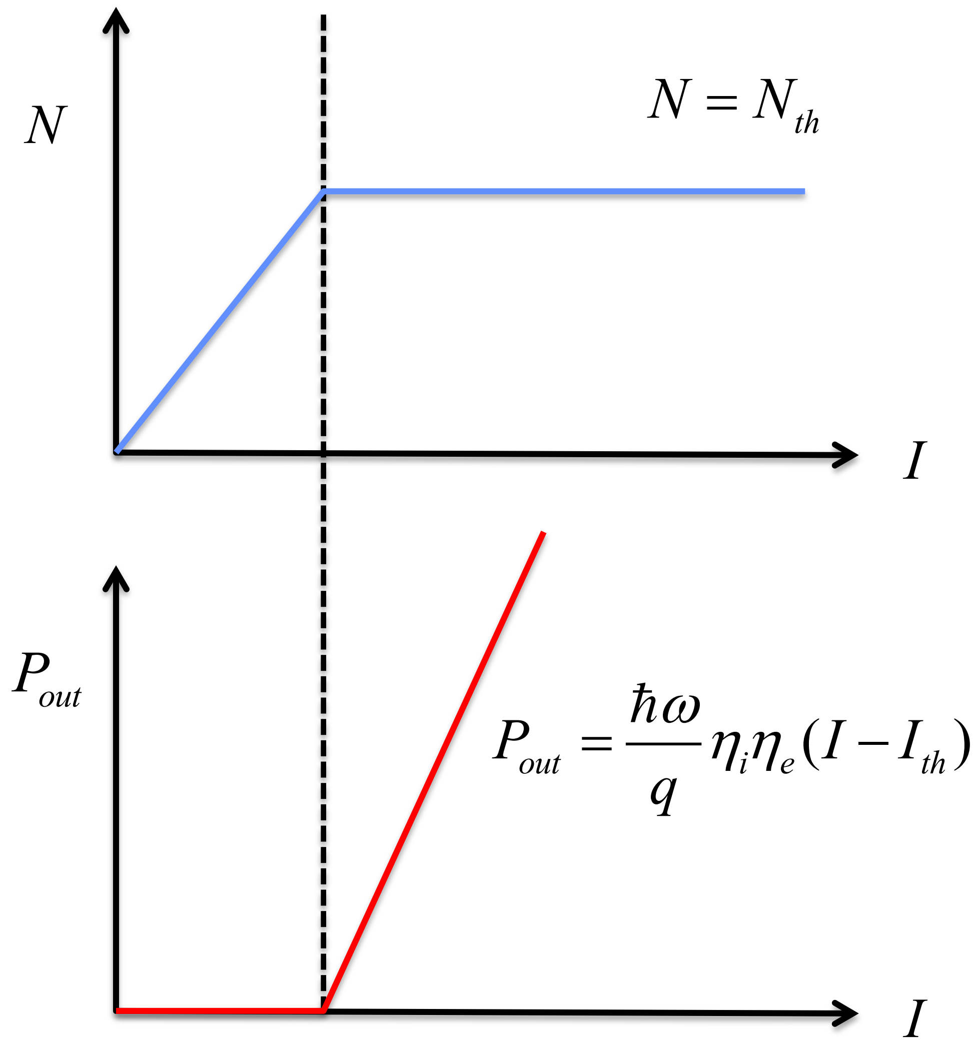
Gain, and therefore carrier concentration, are clamped at threshold values:

$$S = \frac{1}{\Gamma v_g g_{th}}\left(\eta_i \frac{J}{qd} - R_{SRH} - R_{Auger}\right)$$

$$S = \frac{1}{\Gamma v_g g_{th}} \frac{\eta_i \left(J - J_{th}\right)}{qd}$$

$$P_{stim} = (\hbar\omega)(v_g \alpha_m)(S)(V_{cav}) = \eta_i \left(\frac{\alpha_m}{\alpha_m + \alpha_i}\right)\frac{\hbar\omega}{q}(I - I_{th})$$

Simplified Analysis: Steady State



Full Analysis

Rate equation for carriers in active region

$$\frac{dn}{dt} = \eta_i \frac{J}{qd} - R_{SRH} - R_{sp} - R_{stim} - R_{Auger}$$

(steady state)

$$\boxed{J = \frac{qd}{\eta_i}\left(R_{SRH} + R_{sp} + R_{stim} + R_{Auger}\right)}$$

Rate equation for photons in cavity

$$\frac{dS}{dt} = -\frac{S}{\tau_p} + \Gamma\beta_{sp}R_{sp} + \Gamma R_{stim}$$

Note, recall: $R_{stim} = v_g g S$

(steady state)

$$\boxed{S = \frac{\Gamma\beta_{sp}R_{sp}}{\frac{1}{\tau_p} - \Gamma v_g g}}$$

Below threshold – Spontaneous emission

From before, $R_{sp} = \eta_{sp} \eta_i J/qd$

$$S = \left(\Gamma \beta_{sp} R_{sp}\right) \Big/ \left(1/\tau_p - \Gamma v_g g\right)$$
$$\simeq \tau_p \Gamma \beta_{sp} R_{sp}$$
$$= \tau_p \Gamma \beta_{sp} \eta_i \eta_{sp} \left(J/qd\right)$$

Recall, $\Gamma g_{th} = \dfrac{1}{v_g \tau_p} = \alpha_m + \alpha_i \rightarrow \tau_p = \dfrac{1}{v_g(\alpha_m + \alpha_i)}$

Then, $S = \dfrac{\Gamma \beta_{sp} \eta_{sp} \eta_i}{v_g(\alpha_m + \alpha_i)} \dfrac{J}{qd}$ This is the number of spontaneously emitted photons in the cavity mode

Below threshold – Spontaneous emission

Spontaneous emission power (P_{sp})

= (Photon energy) × (Rate of loss from mirror) × (Photon density) × (Cavity volume)

$$P_{sp} = (\hbar\omega)(v_g \alpha_m)(S)(V_{cav})$$

$$= \hbar\omega v_g \alpha_m \frac{\Gamma \beta_{sp} \eta_{sp} \eta_i}{v_g(\alpha_m + \alpha_i)} \frac{J}{qd} V_{cav}$$

Note:

$$\Gamma \simeq \frac{V_{act}}{V_{cav}} = \frac{(\text{Area})d}{V_{cav}} \rightarrow V_{cav} = \frac{(\text{Area})d}{\Gamma}$$

$$\boxed{P_{sp} = \eta_{sp} \eta_i \left(\frac{\alpha_m}{\alpha_m + \alpha_i} \right) \frac{\hbar\omega}{q} \beta_{sp} I}$$

This is the spontaneous emission power into the cavity mode

At threshold and above threshold

At threshold

Laser threshold condition is reached when gain precisely balances cavity loss. Stimulated emission will still be small with respect to spontaneous emission.

$$\eta_i \frac{J_{th}}{qd} = R_{SRH}(n_{th}) + R_{sp}(n_{th}) + R_{Auger}(n_{th})$$

Above threshold

Above threshold and under steady state operation, the gain must "clamp" at the threshold gain. If this were note the case, the fields in the cavity would grow without bound (thus not achieving steady state). This also implies that the Fermi-level and current density must clamp at the threshold current density.

$$g \simeq g_{th} \quad \text{for } I > I_{th}$$

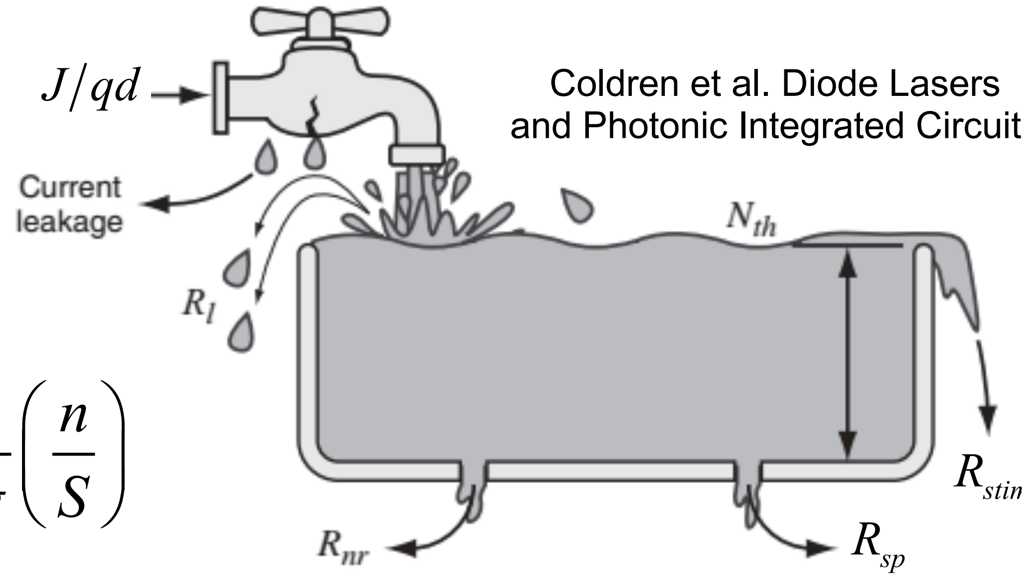
$$n \simeq n_{th} \quad \text{for } I > I_{th}$$

Gain and carrier density clamping

Recall, $S = \dfrac{\Gamma \beta_{sp} R_{sp}}{1/\tau_p - \Gamma v_g g}$

Now, let's derive a stimulated emission lifetime.

$$R_{stim} = \frac{n}{\tau_{stim}} = v_g g S \rightarrow \tau_{stim} = \frac{1}{v_g g}\left(\frac{n}{S}\right)$$



Coldren et al. Diode Lasers and Photonic Integrated Circuits.

We see that there is a negative feedback loop that does not allow the gain to increase beyond the gain threshold. As current is increased and gain approaches the threshold gain, the photon density increases dramatically thus significantly reducing the stimulated emission lifetime. Any additional current gets immediately "used up" by stimulated emission not allowing the carrier density or gain to increase.

The laser at threshold is similar to a filled bathtub. Any additional water spills over the side. Likewise any additional injected carriers will "spill out" as stimulated emission and will not increase the carrier density.

Above threshold

From the carrier density rate equation

$$\eta_i \frac{J}{qd} = R_{SRH}(n_{th}) + R_{sp}(n_{th}) + R_{Auger}(n_{th}) + R_{stim}$$

$$\eta_i \frac{J}{qd} = \eta_i \frac{J_{th}}{qd} + v_g g_{th} S$$

$$\frac{\eta_i (J - J_{th})}{qd} = v_g g_{th} S$$

Plug into the photon density rate equation

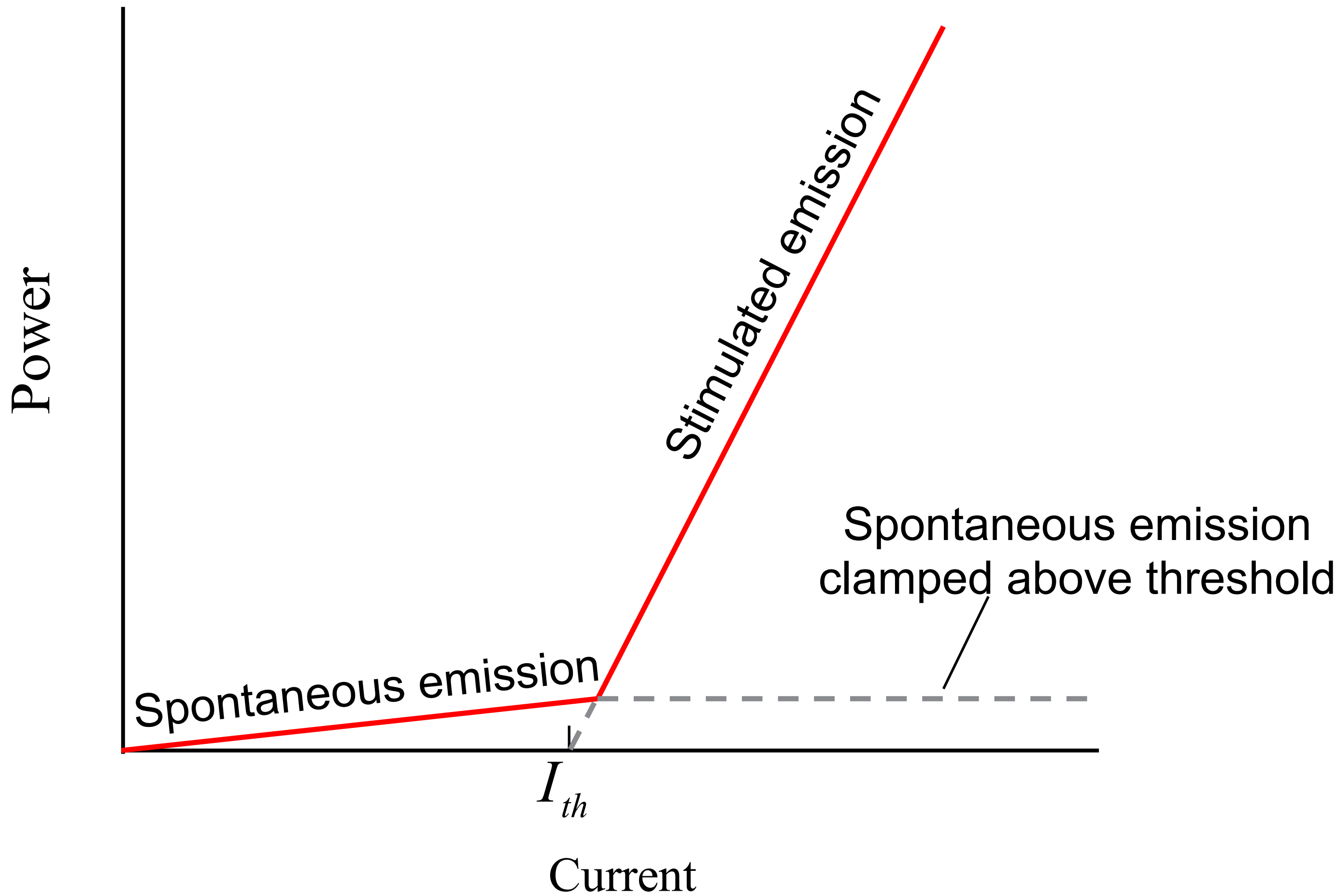
$$\frac{S}{\tau_p} = \Gamma \beta_{sp} R_{sp} + \Gamma v_g g_{th} S \simeq \Gamma v_g g_{th} S$$

$$S = \Gamma \tau_p \frac{\eta_i (J - J_{th})}{qd}$$

$$P_{stim} = (\hbar\omega)(v_g \alpha_m)(S)(V_{cav}) = \eta_i \left(\frac{\alpha_m}{\alpha_m + \alpha_i} \right) \frac{\hbar\omega}{q} (I - I_{th})$$

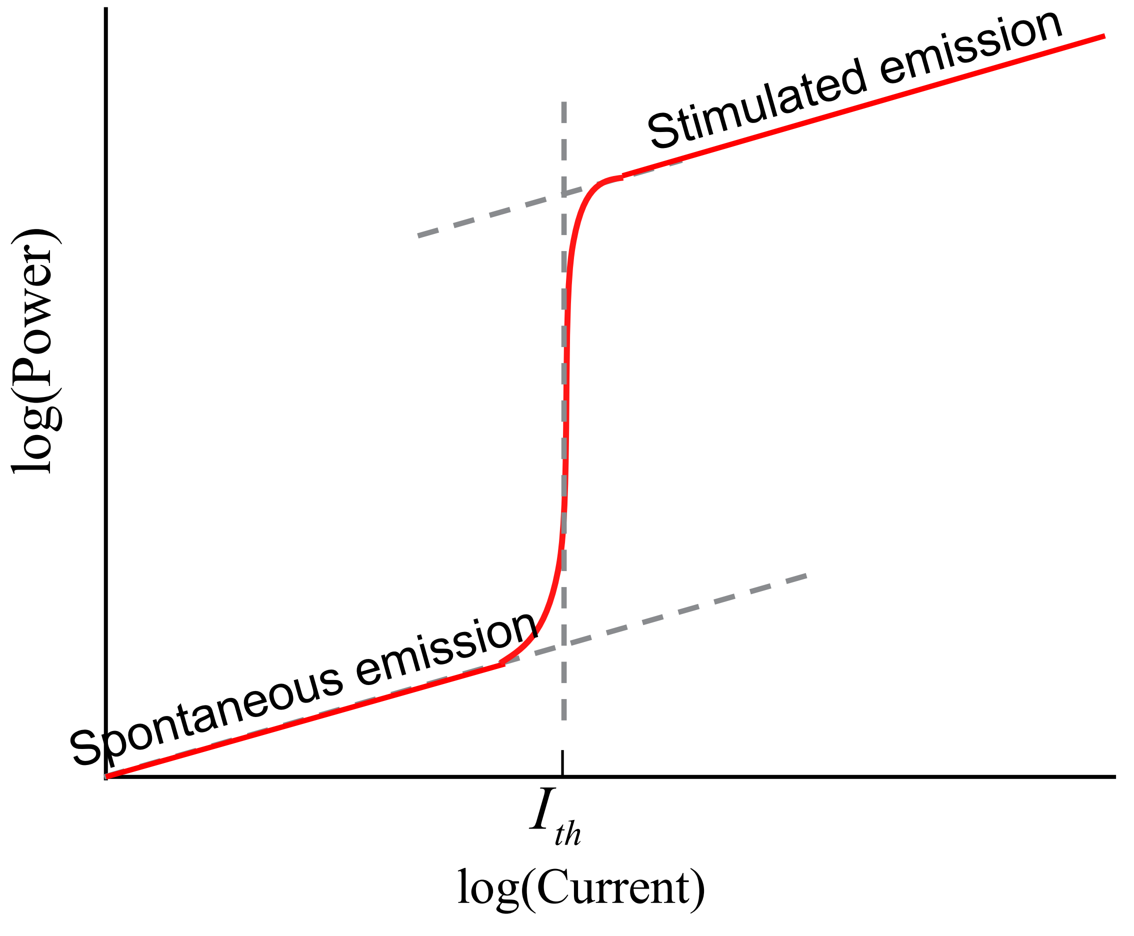
Above threshold (L-I curve)

Above threshold, emitted power is dominated by stimulated emission. Spontaneous emission power clamps at the threshold power.

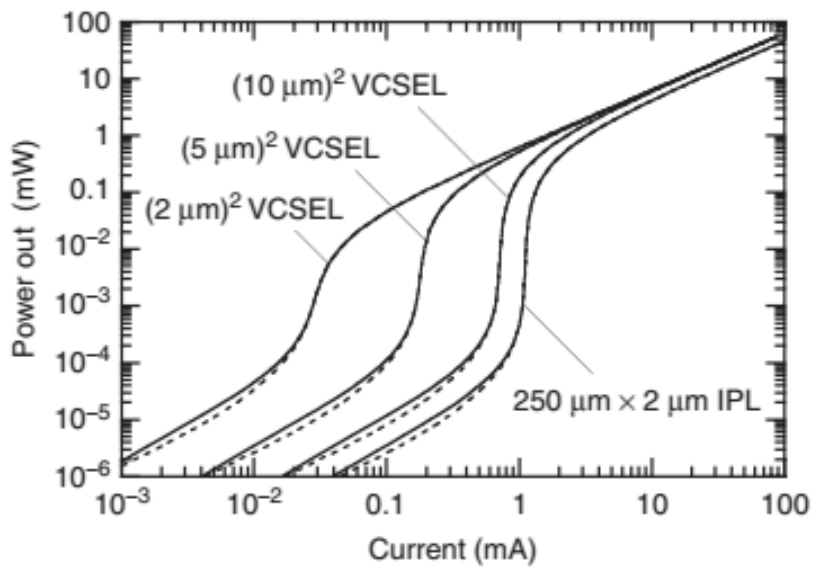


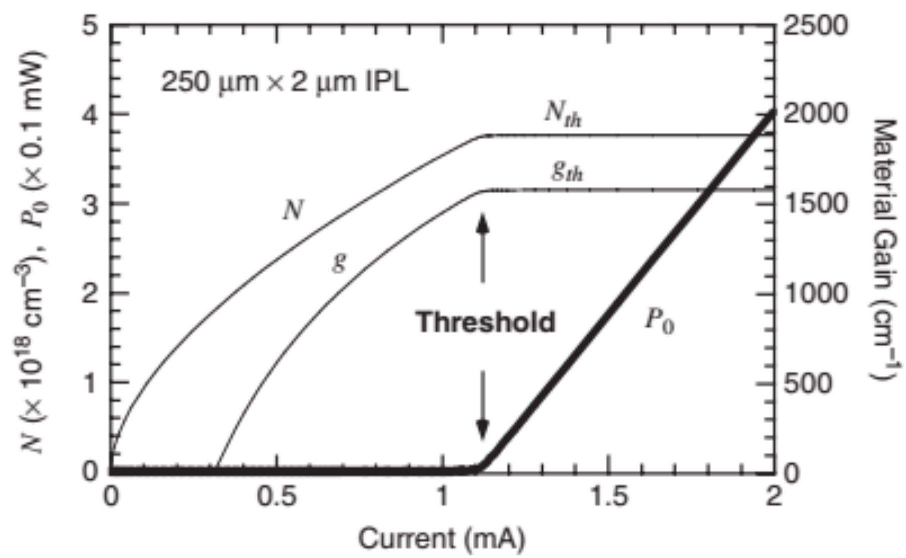
Above threshold (L-I curve)

Plotted on log-log scale the L-I curve has an "S"-like shape.



Above threshold (L-I curve)





Coldren et al. Diode Lasers and Photonic Integrated Circuits.

Differential quantum efficiency

We define the external differential quantum efficiency of the laser as

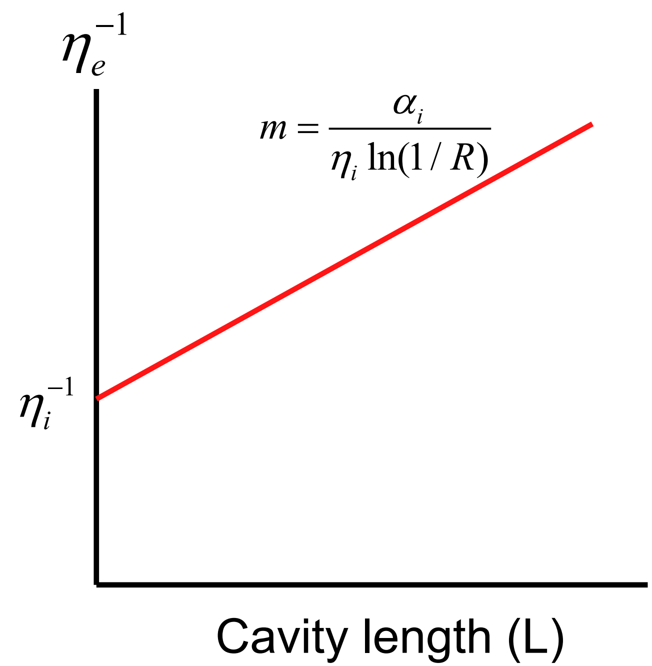
$$\eta_e = \frac{q}{\hbar\omega}\frac{dP}{dI}$$

This is the probability of externally collecting a stimulated photon for an injected electron-hole pair.

$$\eta_e = \eta_i\left(\frac{\alpha_m}{\alpha_m + \alpha_i}\right) = \eta_i \frac{\ln(1/R)}{\alpha_i L + \ln(1/R)}$$

By plotting the inverse of the external differential quantum efficiency as a function of cavity length we can determine cavity loss and injection efficiency from the slope.

$$\frac{1}{\eta_e} = \frac{\alpha_i}{\eta_i \ln(1/R)} L + \frac{1}{\eta_i}$$



Summary

Light emitting diode

$$I = V_{act}\frac{q}{\eta_i}(R_{SRH} + R_{sp} + R_{Auger}) \qquad P_{sp} = \eta_{IQE}\eta_c \frac{\hbar\omega}{q} I$$

Semiconductor laser

<u>Below threshold:</u>

$$I = V_{act}\frac{q}{\eta_i}(R_{SRH} + R_{sp} + R_{Auger})$$

$$P_{sp} = \eta_{sp}\eta_i\left(\frac{\alpha_m}{\alpha_m + \alpha_i}\right)\frac{\hbar\omega}{q}\beta_{sp} I$$

(Spontaneous emission power into lasing mode)

<u>Above threshold:</u>

$$I = V_{act}\frac{q}{\eta_i}\left[(R_{SRH} + R_{sp} + R_{Auger})\Big|_{n=n_{th}} + R_{stim}\right]$$

$$P_{stim} = \eta_i\left(\frac{\alpha_m}{\alpha_m + \alpha_i}\right)\frac{\hbar\omega}{q}(I - I_{th})$$

(Stimulated emission power into lasing mode)