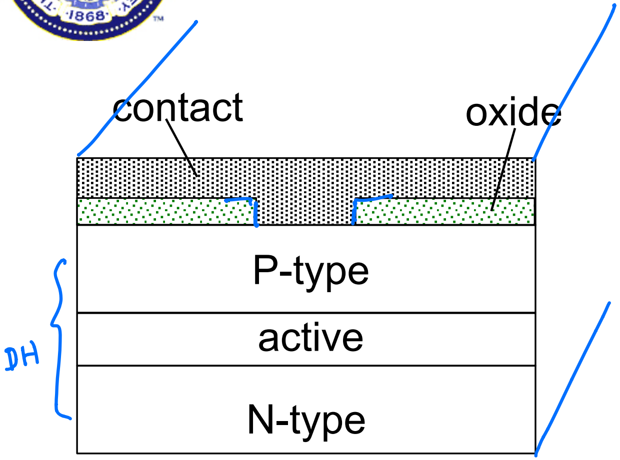
EE 232 Lightwave Devices
Lecture 21: Steady State Response of Semiconductor LEDs and Lasers

Instructor: Ming C. Wu

University of California, Berkeley
Electrical Engineering and Computer Sciences Dept.

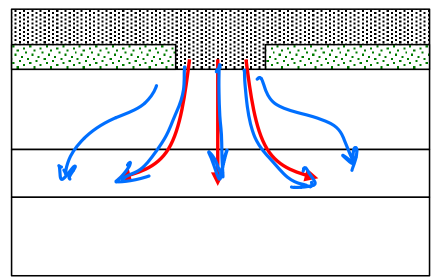
Acknowledgment: some lecture materials are provided by Seth Fortuna

Current and photon confinement



Edge-emitting stripe laser

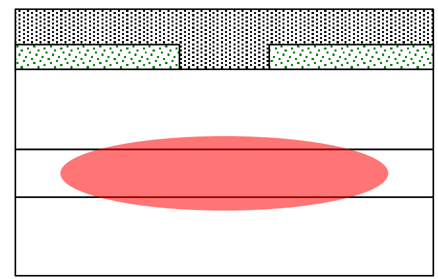
Current confinement



Oxide confines current in lateral direction.

No confinement in lateral direction

Photon confinement



Index contrast confines mode in transverse direction

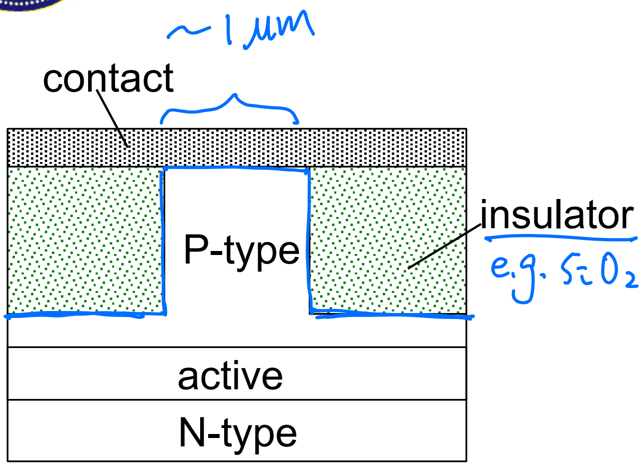
No confinement in lateral direction

No index. guiding

lateral direction

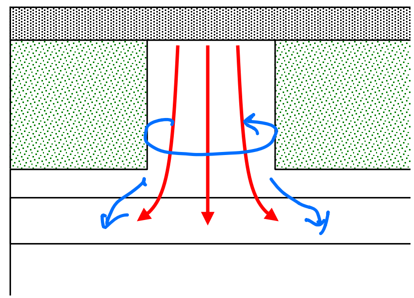
transverse direction

Current and photon confinement



Edge-emitting ridge laser

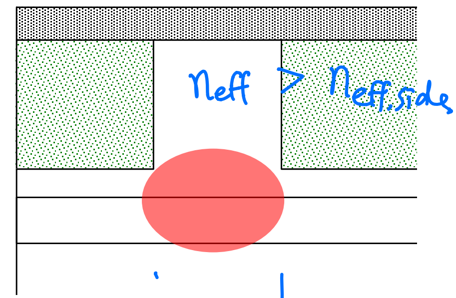
Current confinement



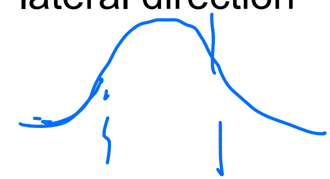
Heterostructure confines current in transverse direction.

Ridge confines current in lateral direction.

Photon confinement

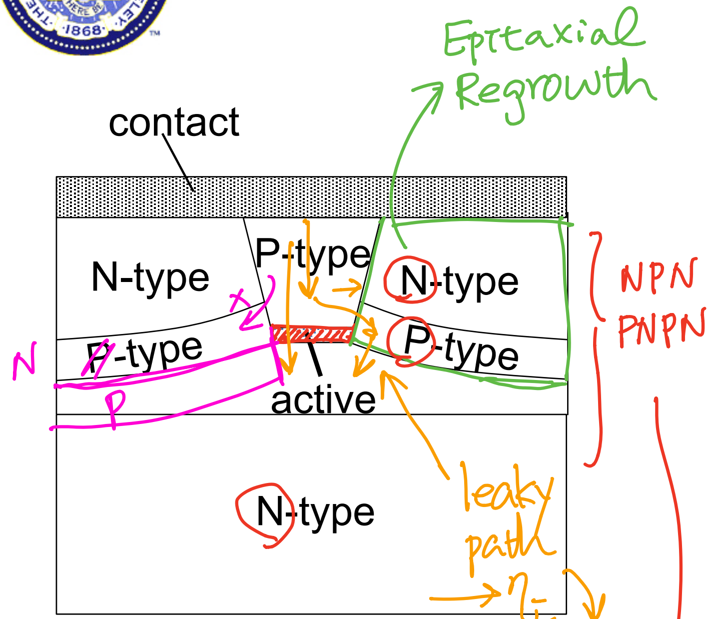


Index contrast confines mode in transverse and lateral direction



Index Guided

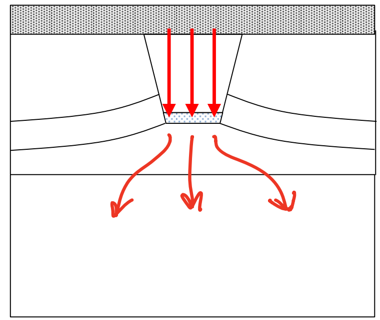
Current and photon confinement



Edge-emitting buried ridge stripe

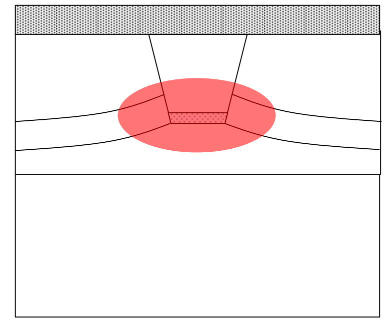
Buried Heterostructure (BH)

Current confinement



Heterostructure and reverse-biased pn junction confines current in transverse and lateral direction.

Photon confinement



Index contrast confines mode in transverse and lateral direction

Index Guided

Alternative for InP

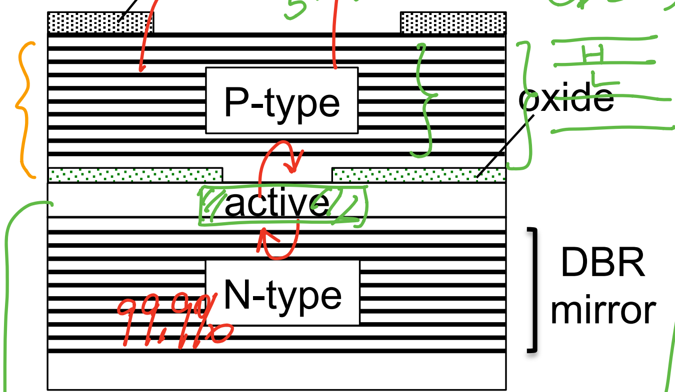
Fe: InP, deep trap → "Semi-Insulating" InP

↳ Better than NPN or PNPN, ① Better insulating ② No leakage ③ Capacitance lower

Vertical Cavity Surface-Emitting Laser (VCSEL)

Current and photon confinement

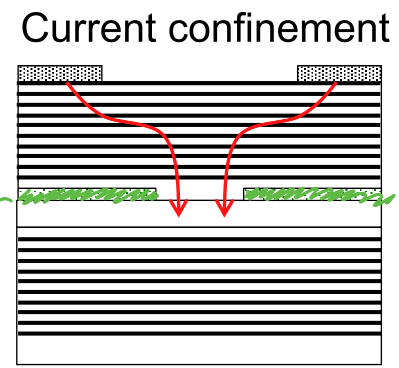
Distributed Bragg Reflector (DBR)



Surface-emitting
oxide aperture laser

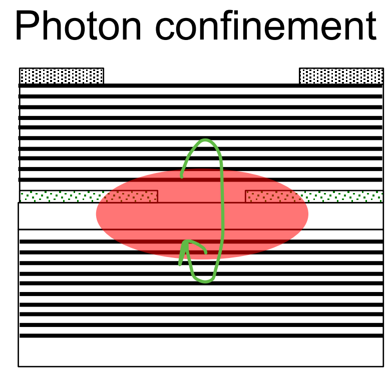
One QW

Oxide
oxidize AlAs



Oxide confines current
in lateral direction.

Heterostructure
confines current
in transverse direction



Index contrast confines
mode in transverse and
lateral direction

Metal: Au, $R \lesssim 96\%$
Ag, $R \sim 98\%$

DBR: $R \sim 99.9\% - 99\%$

$\alpha_m = \frac{1}{2L} \ln \frac{1}{R^2}$

$L \sim 100\ \mu m$ edge

$L \sim 1\ \mu m$ VCSEL

lower power → Small aperture: single mode

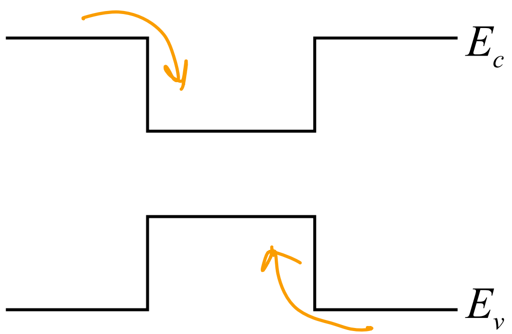
higher power → large " → Multi-modes

$P_{out} = \eta_i \eta_{sp} I \cdot \left(\frac{\hbar\omega}{q}\right) \cdot \frac{\alpha_m}{\alpha_m + \alpha_i}$

Current and photon confinement

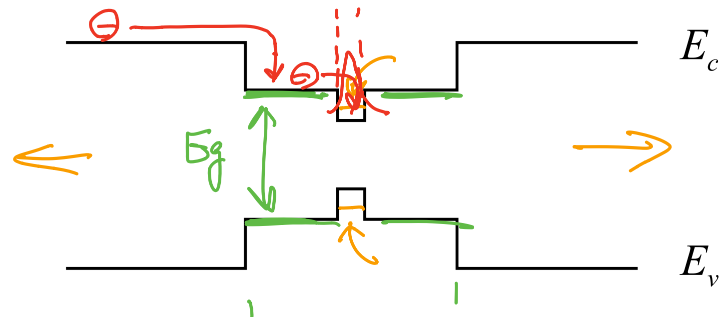
DH

QW, ≲ 10 nm



Double heterostructure

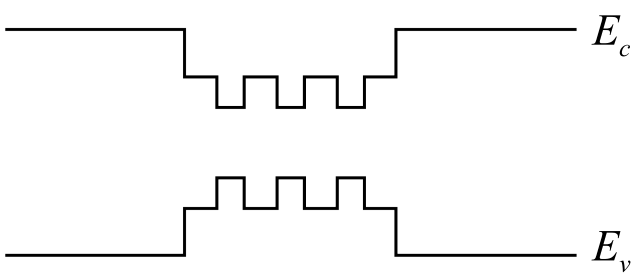
↳ Slab Waveguide



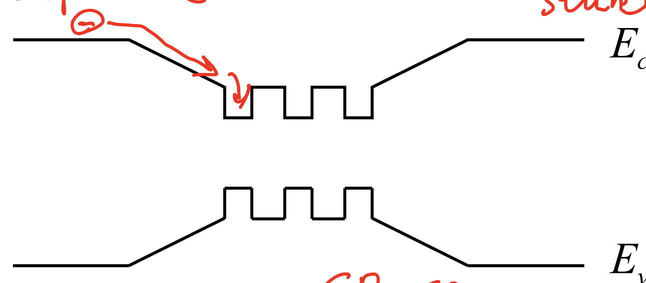
Single quantum well separate confinement heterostructure

(SCH)

separate confinement hetero-structure



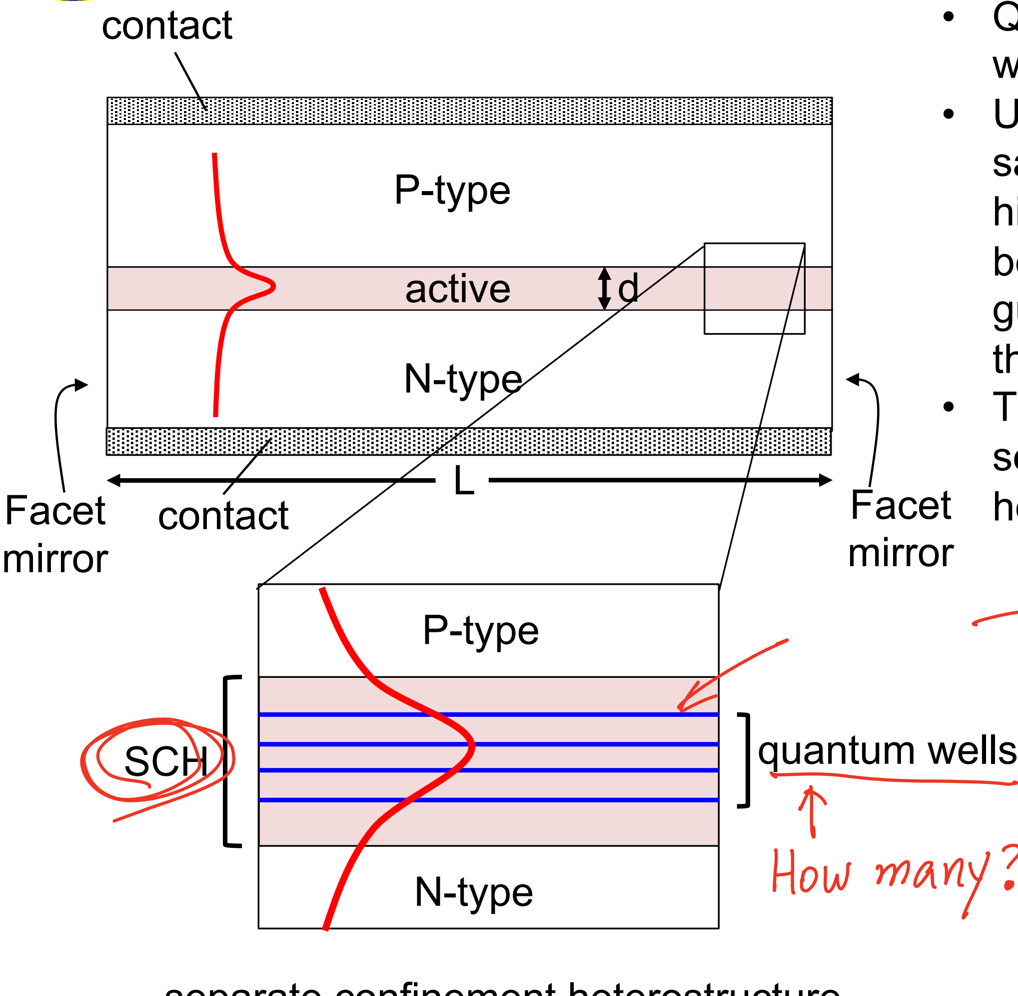
Multiple quantum well (MQW) separate confinement heterostructure



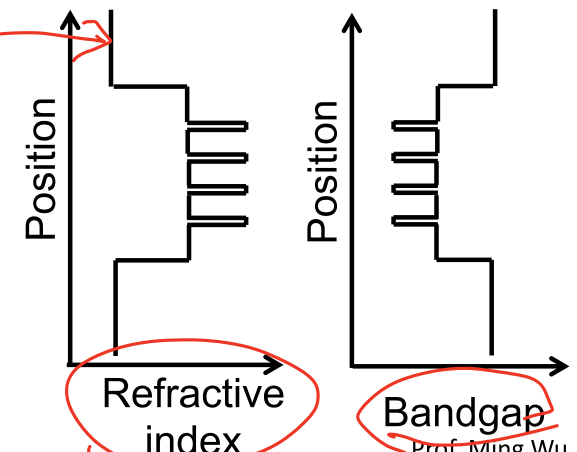
GR SCH

Graded index separate confinement heterostructure (GRINSCH)

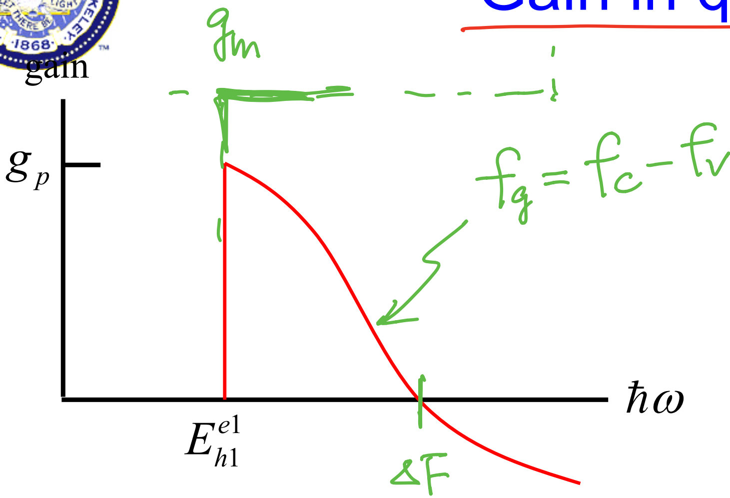
Edge-Emitting Quantum well laser



- Quantum well(s) can only very weakly confine a mode.
- Usually, quantum wells are sandwiched inside another higher bandgap material that can be engineered to improve mode guiding and electron injection into the quantum well(s).
- This type of structure is called a separate confinement heterostructure (SCH).



Gain in quantum well



Peak gain occurs at the bandedge

$$g_p = g_m(f_c[\hbar\omega = E_{h1}^{e1}] - f_v[\hbar\omega = E_{h1}^{e1}])$$

Assuming only subband is filled in conduction and valence bands we can write an approximate expression for the peak gain

Recall,

$$n = n_c \ln\left(1 + \exp[(F_c - E_g - E_{e1})/kT]\right)$$

$$p = n_v \ln\left(1 + \exp[(E_{h1} - F_v)/kT]\right)$$

$$n_c = \frac{m_e^* kT}{\pi\hbar^2 L_z} \qquad n_v = \frac{m_h^* kT}{\pi\hbar^2 L_z}$$

The Fermi functions can be written in terms of the carrier density

$$f_c(\hbar\omega = E_{h1}^{e1}) = \frac{1}{1 + \exp\left[\left(E_g + E_{e1} - F_c\right)/kT\right]} = 1 - \exp(-n/n_c)$$

$$1 - e^{-\frac{n}{n_c}}$$

$$f_v(\hbar\omega = E_{h1}^{e1}) = \frac{1}{1 + \exp\left[\left(E_{h1} - F_v\right)/kT\right]} = \exp(-p/n_v)$$

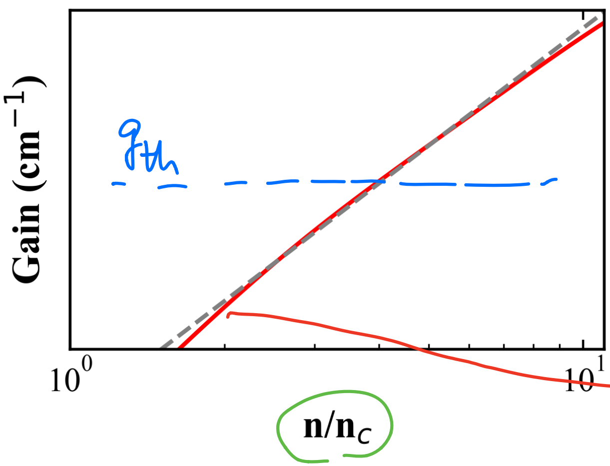
$$e^{-\frac{p}{n_v}}$$

Then, $\boxed{g_p = g_m\left[1 - \exp(-n/n_c) - \exp(-p/n_v)\right]}$

Gain in quantum well

Physics

$$g_p = g_m\left[1-\exp(-n/n_c)-\exp(-p/n_v)\right]$$



The plot of peak gain vs. carrier concentration (red line) is approx. linear on a semi-log plot, therefore a simpler approximate expression is often used (dashed gray curve).

$$\boxed{g_p = g_0 \ln[n/n_0]}$$

logrithmic gain "mode"

As discussed previously, the current density can be written in terms of a polynomial of the carrier density (e.g. $J \propto n^2$ if spontaneous emission dominates). Therefore, we can write a similar approximate expression for peak gain in terms of carrier density.

$$\boxed{g_p = g_0 \ln[J/J_0]}$$

Note: g_0 are not the same in both expressions

Quantum well laser - threshold gain

$$g_w = g_0 \ln\left(\frac{J_w}{J_0}\right)$$

$$n_w \Gamma_w g_w = \alpha_i + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

g_w: quantum well threshold gain

J_w : threshold quantum well current density

n_w: number of quantum wells in active region

Γ_w : fraction of mode in quantum well

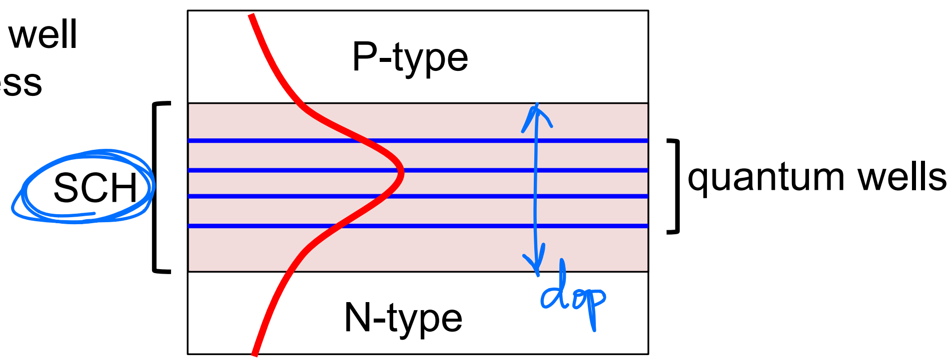
$$\Gamma_w = \Gamma \frac{L_z}{d_{op}}$$

quantum well thickness

optical confinement factor

of SCH

effective width of optical mode

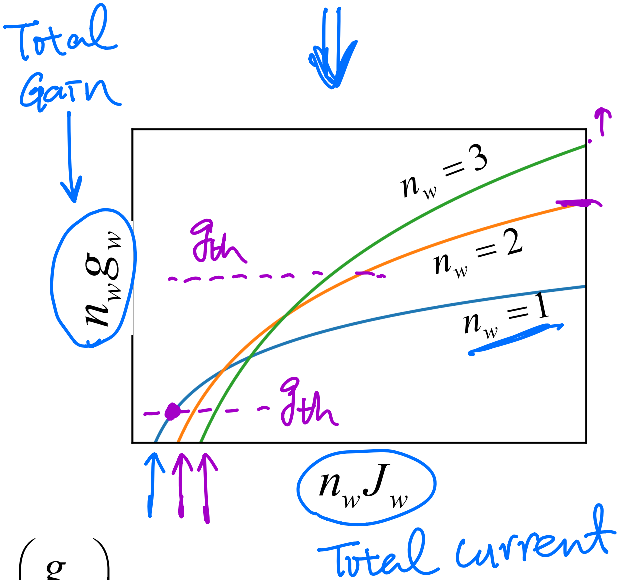


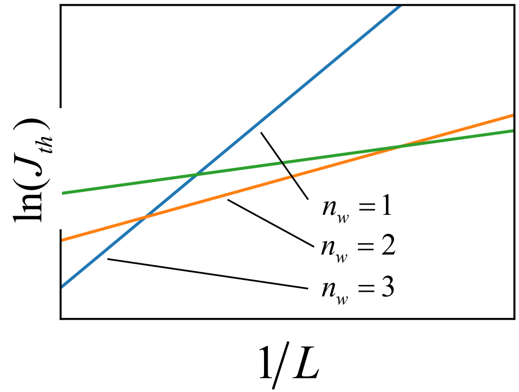
Threshold current and gain in active region with multiple quantum wells

$$n_w g_w = n_w g_0 \ln\left(\frac{n_w J_w}{n_w J_0}\right)$$

Note: $J_{th} = \dfrac{n_w J_w}{\eta_i}$

threshold current at device terminals





Then: $$J_{th} = \frac{n_w J_0}{\eta_i} \exp\left(\frac{g_w}{g_0}\right)$$

$$J_{th} = \frac{n_w J_0}{\eta_i} \exp\left(\frac{1}{n_w \Gamma_w g_0}\left[\alpha_i + \frac{1}{2L}\ln\frac{1}{R_1 R_2}\right]\right)$$

$$\boxed{I_{th} = \frac{wLn_w J_0}{\eta_i} \exp\left(\frac{1}{n_w \Gamma_w g_0}\left[\alpha_i + \frac{1}{2L}\ln\frac{1}{R_1 R_2}\right]\right)}$$

w: active region width

L : active region length

Active region optimization

Optimize cavity length (L) or a fixed number of quantum wells

$$\frac{\partial I_{th}}{\partial L} = 0 \rightarrow \boxed{L_{opt} = \frac{1}{2}\frac{1}{n_w \Gamma_w g_0} \ln\left(\frac{1}{R_1 R_2}\right)}$$

Optimize number of quantum wells (n_w) for a fixed cavity length

$$\frac{\partial I_{th}}{dn_w} = 0 \rightarrow \boxed{n_{opt} = \frac{1}{\Gamma_w g_0}\left(\alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right)}$$

For a general cavity, $\frac{1}{\tau_p v_g} = \left(\alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2}\right)$

$$Q = \omega_0 \tau_p$$

therefore, $\boxed{n_{opt} = \frac{1}{\Gamma_w g_0 v_g} \frac{\omega_0}{Q}}$

Additional details on the "ABC" approximation

Recombination rate in LED or Laser at or below threshold:

$$R = R_{SRH} + R_{sp} + R_{Auger}$$

$$\simeq An + Bn^2 + Cn^3 \quad \text{"ABC" approximation}$$

The ABC approximation is widely used to estimate recombination rates in LEDs and lasers (at or below threshold). Although strictly speaking, it is valid only when Boltzmann statistics are valid so some care needs to be applied when using the approximation.

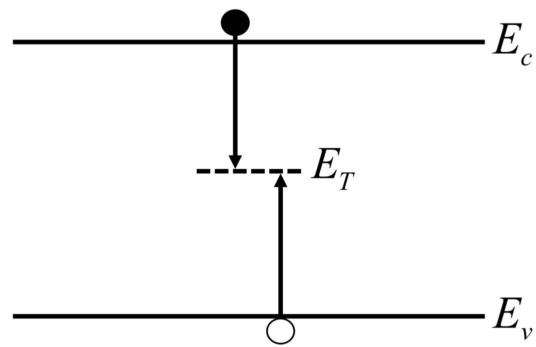
We have already proved that the spontaneous emission rate has an n^2 dependence (when Boltzmann statistics apply). See the previous lecture on spontaneous emission.

Let's look at the Shockley-Reed-Hall and Auger rates.

Shockley-Reed-Hall recombination

The derivation of the SRH rate is found in many basic semiconductor textbooks.

$$R_{SRH} \simeq \frac{n_0 \delta n + p_0 \delta n + \delta n^2}{C_p^{-1}(n_0 + n_{ds} + \delta n) + C_n^{-1}(p_0 + p_{ds} + \delta n)}$$



$n = n_0 + \delta n$

$p = p_0 + \delta p$

$\delta_n = \delta_p$

n_{ds} and p_{ds} are the electron and hole concentrations when the Fermi level is at the defect state energy

$C_n = \sigma_n v_{n,th} N_{ds}$
$C_p = \sigma_p v_{p,th} N_{ds}$
] Capture rates

σ_n
σ_p
] Capture cross-sections

v_n
v_p
] Thermal velocity

N_{ds} : defect density

We see that in general we cannot write

$$R_{SRH} = An$$

But, we can do so if we restrict our analysis to a "low-injection" or "high-injection" regime.

Low-injection and high-injection regime

Active region materials have a background doping due to unintentional doping impurities introduced during growth. Let's assume our active region is unintentionally doped p-type.

$p_0 \gg \delta n$ Low-injection regime

$$R_{SRH} = \frac{n_0 \delta n + p_0 \delta n + \delta n^2}{C_p^{-1}(n_0 + n_{ds} + \delta n) + C_n^{-1}(p_0 + p_{ds} + \delta n)} \cong C_n \delta_n \simeq A_{low} n$$

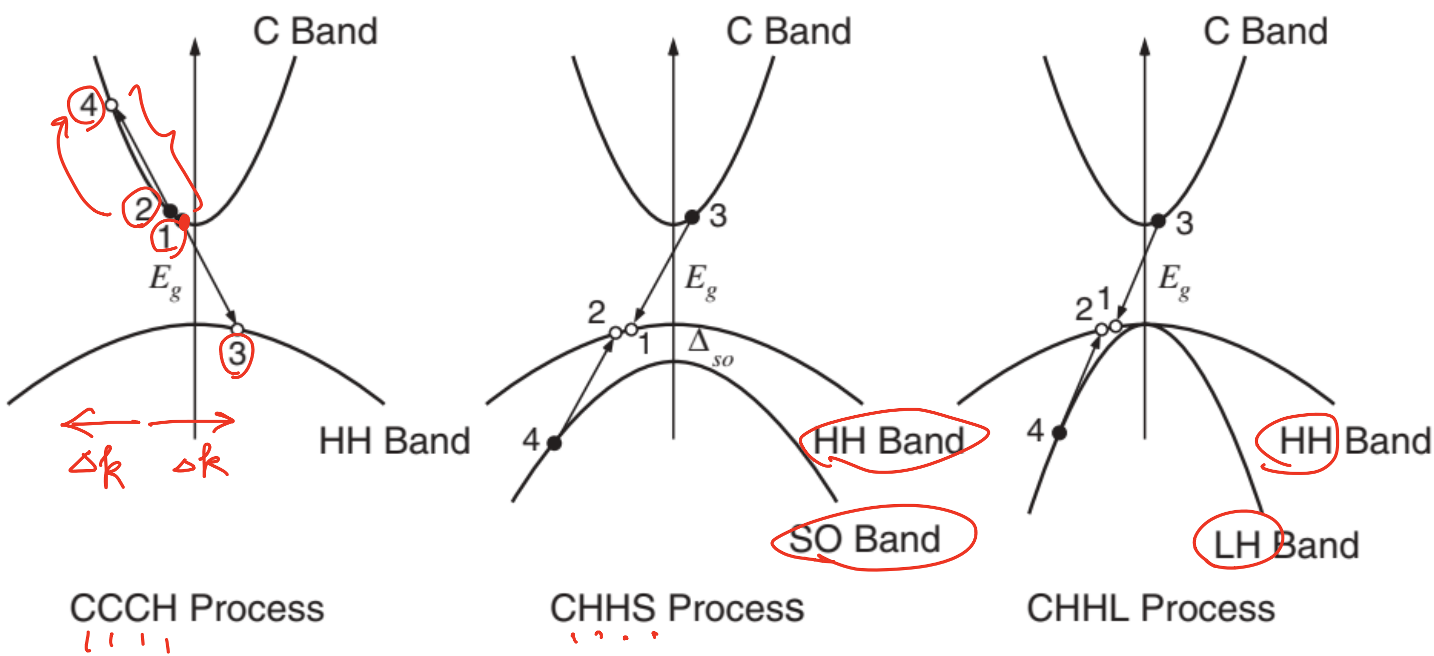
$\delta_n \gg p_0$ High-injection regime

$$R_{SRH} \cong \frac{C_n C_p}{C_n + C_p} \delta_n \simeq A_{high} n$$

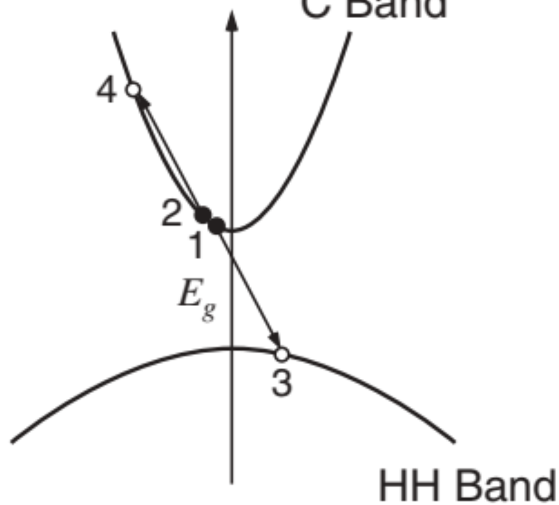
We see that we can write $R_{SRH} = An$ so long we stay in one of the two regimes. If the electron capture is the rate-limiting step (for p-type material), then the A coefficient will be identical in both regimes.

Auger recombination

Electron recombines with hole and gives up excess energy to another carrier instead of releasing a photon. Several different Auger processes are possible (as shown below). Often there is a material-dependent dominant process.



Auger recombination



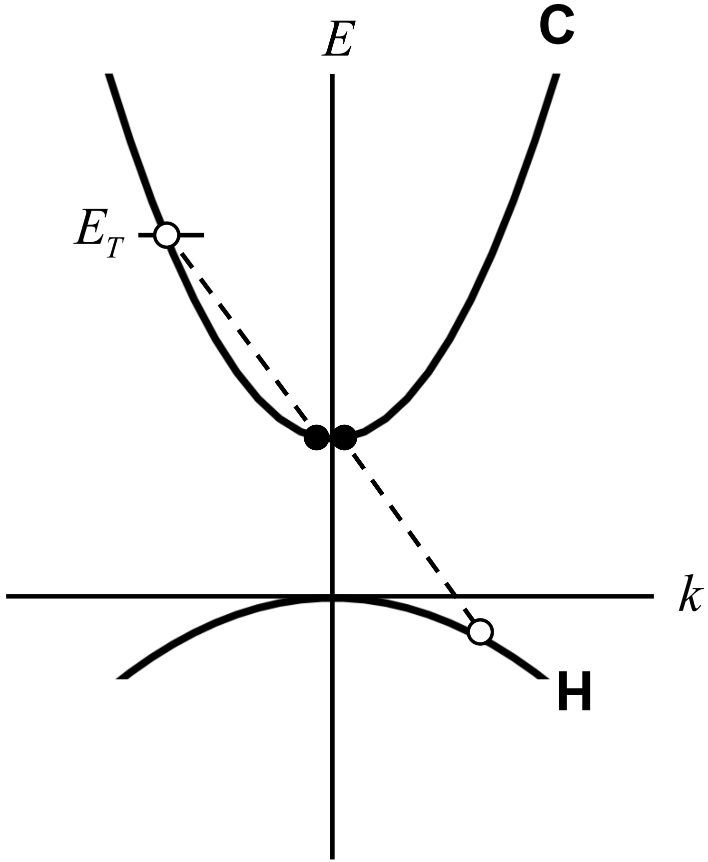
The CCCH Auger rate is given by

$$
\begin{aligned}
R_{Auger} &= C_0 f_1 f_2 (1-f_3)(1-f_4) \\
&= C_0 \exp\left[\frac{F_c - E_1}{kT}\right] \exp\left[\frac{F_c - E_2}{kT}\right] \exp\left[\frac{E_3 - F_v}{kT}\right] (1) \\
&= C_0 \frac{n^2 p}{(kT)^2 N_c^2 N_v} \exp\left[\frac{E_g - E_4}{kT}\right]
\end{aligned}
$$

$$\boxed{R_{Auger} = Cn^2 p = Cn^3 \quad \text{for } n = p}$$

Very likely that State 4 is empty since it is well beyond the bandedge

Auger recombination



Energy and momentum conservation needs to be simultaneously conserved. This sets a threshold value for E_4 which we call E_T. Materials with small E_T will have large Auger rates since

$$R_{Auger} \propto \exp(-E_T / kT)$$

E_T is related to the curvature of the bands through

$$E_T = \frac{2m_e^* + m_h^*}{m_e^* + m_h^*} E_g = aE_g$$

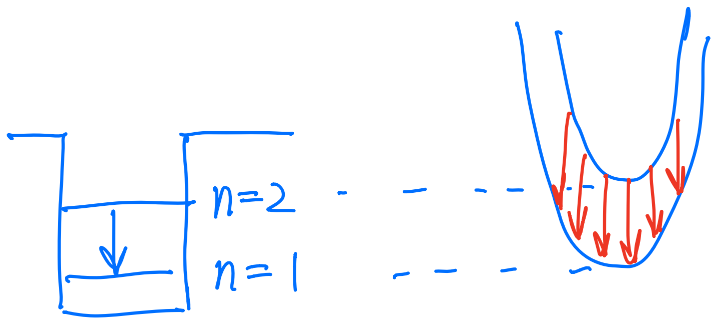
the value of *a* is approximately unity for III-V semiconductors therefore,

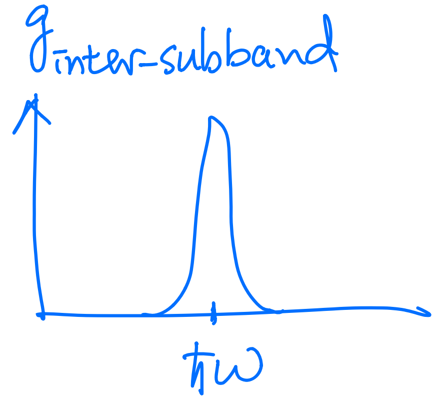
$$R_{Auger} \propto \exp(-E_g / kT)$$

Auger recombination is higher in low bandgap materials.

Tensile strained QW
to reach higher gain
at the expense high N_{tr}
higher power consumption

Quantum Cascaded Laser:

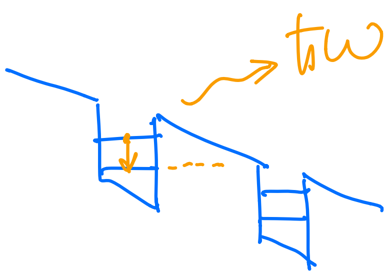


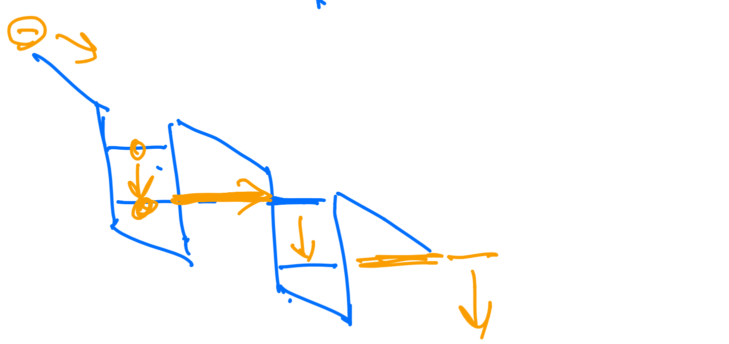


λ: 5um, $\underline{\underline{10 um}}$

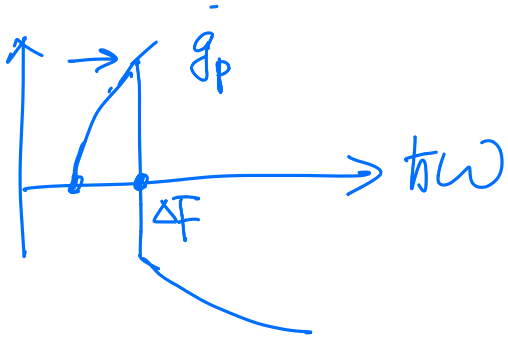
↑

Blackbody radiation, $T \sim 300K$





2017 Final 2(b)

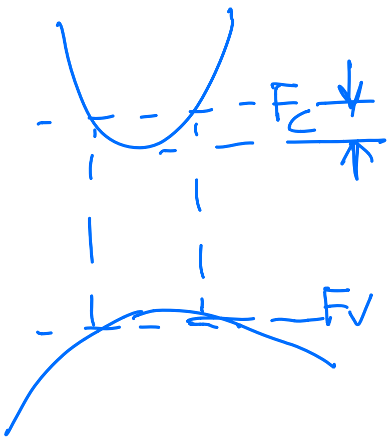


$$g_p = 100 \cdot \sqrt{\underset{\substack{\uparrow \\ =1.25\,eV}}{\hbar\omega} - E_g}$$

$\leftarrow$ 1eV

Calculate $\hbar\omega = 1.25eV = \Delta F = F_c - F_v$

$$n = n(F_c) = N_c \frac{4}{3\sqrt{\pi}}\left(\frac{F_c - E_c}{k_B T}\right)^{3/2}$$

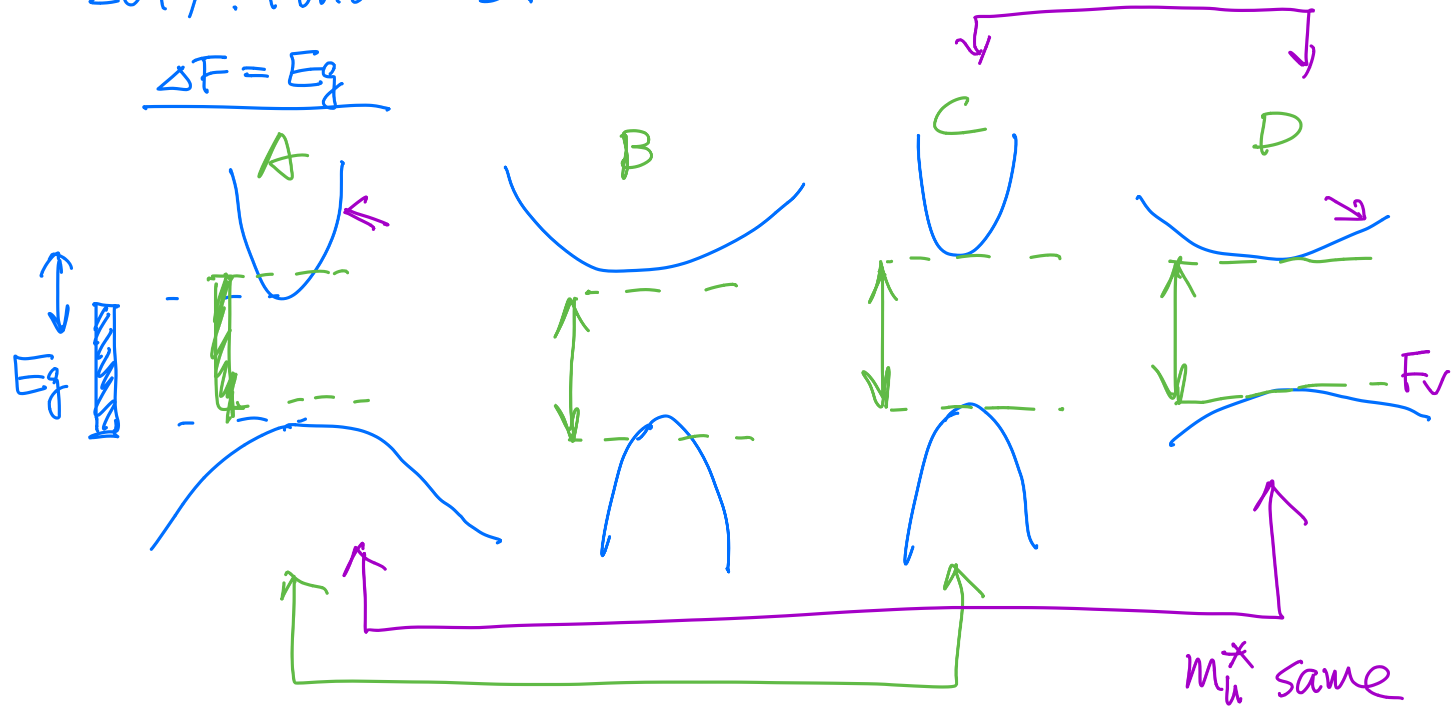


$$(\hbar\omega - E) \times \frac{m_n^*}{m_e^* + m_h^*} = F_c - E_c$$

Q: $\longleftrightarrow$ g_{th}

2017. Final 3.

$\Delta F = E_g$



m_h^* same

$F_{v,D}$ lower

$n_{tr,D}$ larger

Same m_e^*

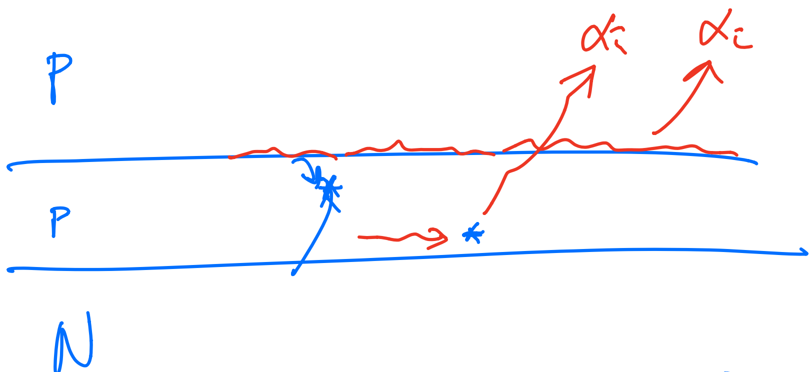
$F_{c,A} > F_{c,C} \Rightarrow n_{tr,A} > n_{tr,c}$

$\boxed{n=P}$ undoped. light doping

heavily doped

p-doped $\quad n + N_A = P_0 + P$

dominate determination of F_v



reduce η_i

$$g(\hbar\omega) = -\alpha(\hbar\omega) = C_0 \cdot |\hat{e}\cdot\vec{P}_{cv}|^2 \cdot \rho_r(\hbar\omega - E_g)\cdot(f_c - f_v)$$

Transparency $\quad f_c = f_v \Rightarrow \Delta F = E_g$

Net gain $\quad f_c > f_v \quad \Delta F > E_g$

Stimulated emission

$$R_{stim} = v_g \cdot g \cdot S$$